Analytical properties of autonomous systems controlled by extended time-delay feedback in the presence of a small time delay mismatch P47

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The application of the delayed feedback control (DFC) or extended DFC (EDFC) algorithms to dynamical systems requires the knowledge of the period of unstable periodic orbit (UPO). For autonomous systems, this period is a priori unknown. An analytical treatment of this problem was first attained by W. Just et. al. [1]. The authors derived an analytical expression for the period of stabilized orbit depending on the delay time in the case of a small delay mismatch. Starting from the autonomous system subject to the DFC $\dot{\mathbf{x}}(t) = \mathbf{F}\left(\mathbf{x}(t), K\left\{g[\mathbf{x}(t)] - g[\mathbf{x}(t-\tau)]\right\}\right)$ (here $\mathbf{x}$ denotes the phase space variables, $g[\mathbf{x}]$ is the measured scalar quantity, τ is the delay time, and K is the control amplitude) they obtained an expression for the period Θ of the periodic solution of the controlled system as an expansion with respect to the small mismatch $\tau - T$ (here T is the period of unstable periodic orbit of the free system):

$$\Theta(K,\tau) = T + \frac{K}{K-\kappa}(\tau - T) + \mathcal{O}((\tau - T)^2), \tag{1}$$

where κ denotes a system parameter which captures all of the details concerning the coupling of the control force to the system. In this work, we present more general result than presented in [1]. Our approach is based on the phase reduction method adapted to the system with time delay [2]. We start from the autonomous system controlled by the EDFC:

$$\dot{\mathbf{x}}(t) = \mathbf{F}\left(\mathbf{x}(t), K\left\{(1-R)\sum_{n=1}^{\infty} R^{n-1} g[\mathbf{x}(t-n\tau)] - g[\mathbf{x}(t)]\right\}\right) \tag{2}$$

The main idea of our approach is based on the splitting of the control force in non-mismatched and mismatched components [3]. The non-mismatched component stabilizes the UPO while the mismatched component induces a small perturbation that can be treated by the phase reduction theory adapted to time-delay systems. As a result we derive a more general expression for the period of the controlled system:

$$\Theta(K,\tau) = T + \frac{K}{K-\kappa(1-R)}(\tau - T) + \mathcal{O}((\tau - T)^2). \tag{3}$$

Another advantage of our approach is that we show how the parameter κ depends on the measured scalar quantity $g[\mathbf{x}]$. The results are important for the practical implementation of the EDFC algorithm, since they facilitate the determination of the unknown period in experiments.

[1] W. Just et al. *Phys. Rev. Lett.* **81(3)** 562-565 (1998).
[2] V. Novičenko and K. Pyragas. *Physica D* **241** 1090-1098 (2012).
[3] V. Novičenko and K. Pyragas. *Phys. Rev. E* **86** 026204 (2012).