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PHASE LEAD SYNCHRONIZATION OF CHAOTIC OSCILLATORS

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FIZINIŲ IR TECHNOLOGIJOS MOKSLŲ CENTRAS

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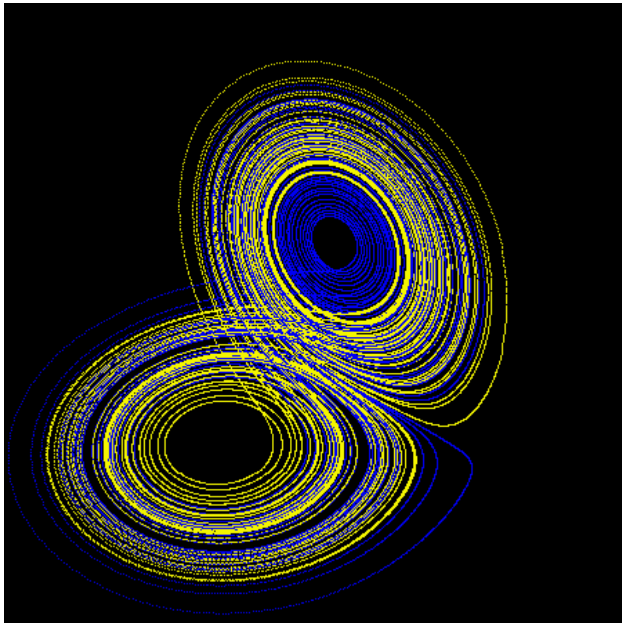
Outline

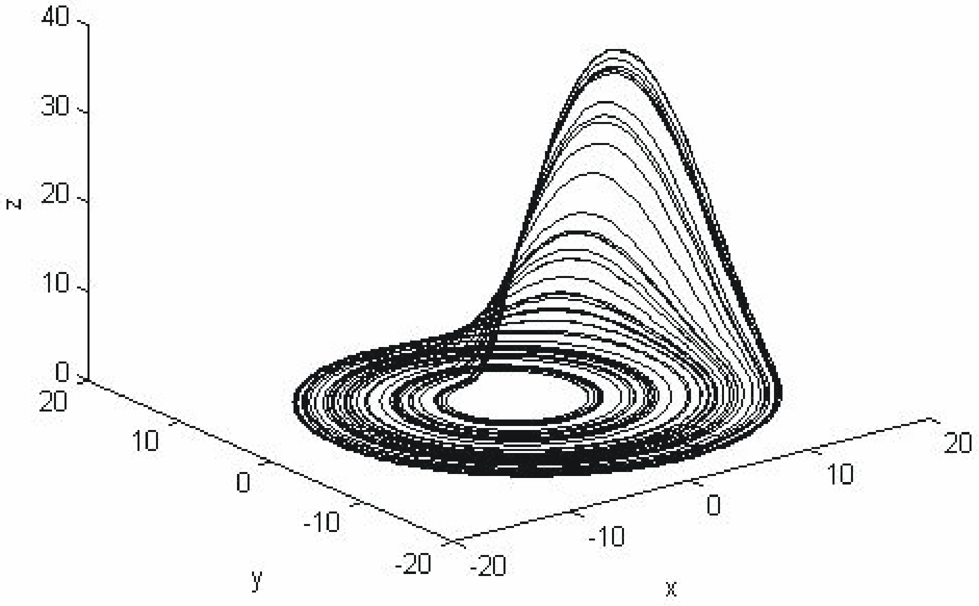
- Prediction of chaotic dynamics
- Phase lead synchronization
  - a simple model
  - Rössler systems
- From phase lead to anticipating synchronization

Chaotic systems

Synchronization and control (1990)

Prediction (2000)





First algorithm

Globally stable solution

$$\mathbf{r}_2(t) = \mathbf{r}_1(t+\tau)$$

$$\dot{\mathbf{r}}_1(t) = -\alpha\,\mathbf{r}_1(t) + \mathbf{f}\left(\mathbf{r}_1(t-\tau)\right) \quad \text{drive system}$$

$$\dot{\mathbf{r}}_2(t) = -\alpha\,\mathbf{r}_2(t) + \mathbf{f}\left(\mathbf{r}_1(t)\right) \quad \text{response system}$$

Disadvantages

A driver has to possess an internal delay term

Fixed time of prediction = driver's delay time

Voss (2000)

Second algorithm

Delay- external parameter

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right),$$ **drive system without delay**

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + \mathbf{K}[\mathbf{r}_1(t) - \mathbf{r}_2(t-\tau)]$$ $$\mathbf{K} = \mathrm{k} \bullet (1,1,1)$$

$$\mathbf{r}_2(t) = \mathbf{r}_1(t+\tau)$$

The solution is stable only for some definite values of delay time and the coupling strength

Voss (2000)

Our former aim was to increase the prediction time

Compensation of the delay

Anticipating synchronization

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + \mathbf{K}[\mathbf{r}_1(t) - \mathbf{r}_2(t-\tau)]$$

stabilizing term with delay

Delayed feedback control method

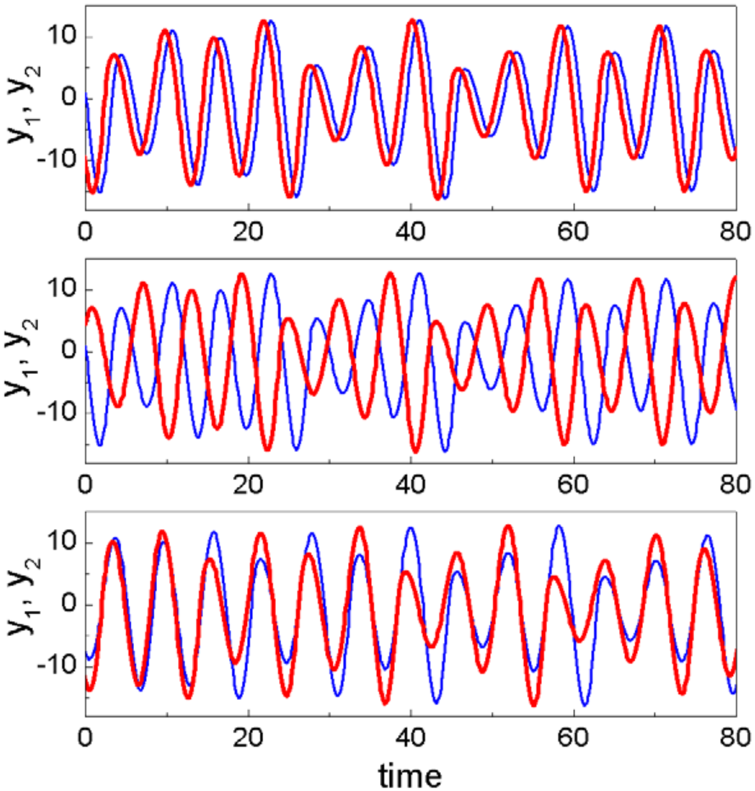
$$\dot{\mathbf{r}}(t) = \mathbf{f}\left(\mathbf{r}(t)\right) - \mathbf{K}[\mathbf{r}(t) - \mathbf{r}(t-\tau)]$$

stabilizing term without delay

Phase lag compensating coupling (PLCC)

The coupling matrix **K** is constructed such that the delay in the response system is compensated

Prediction of the dynamics of chaotic Rössler system



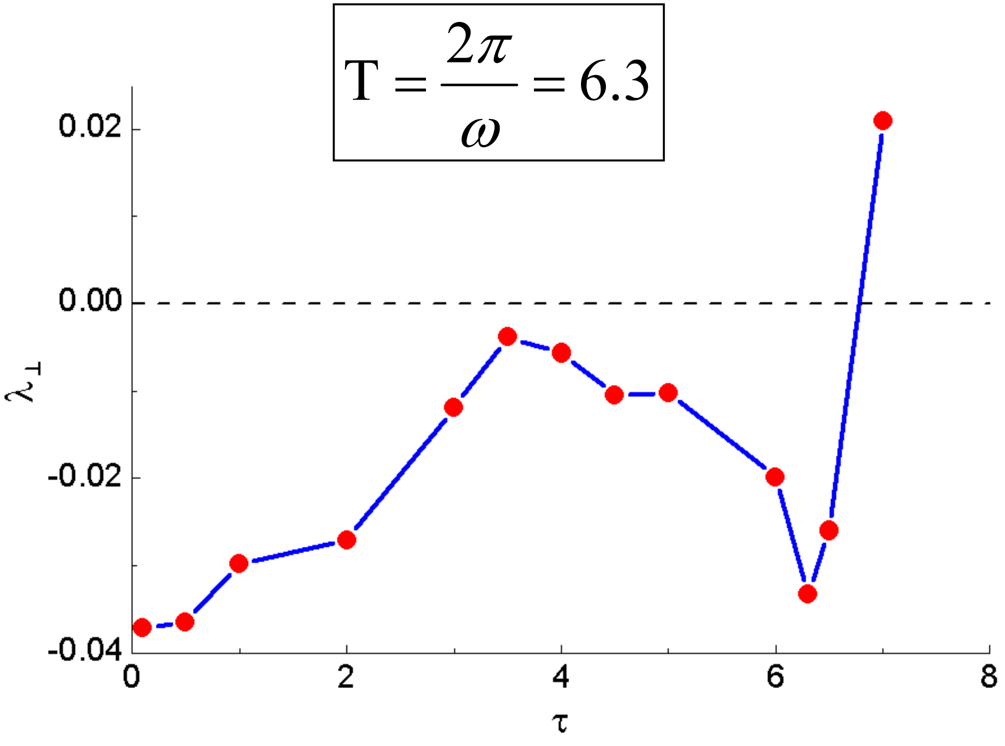
	τ
D	0.9
PLCC	3.6
EPLCC	6.3

The drive system is without delay

The response system obtains the delay term.

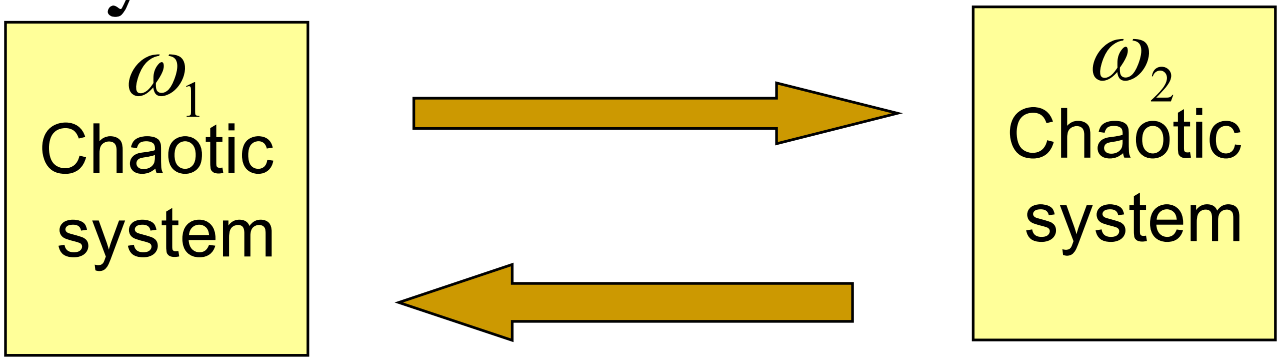
K. Pyragas, T. Pyragienė, Phys. Rev. E **78,** 046217 (2008), Phil. Trans. R. Soc. A **368**, 305 (2010)

Lyapunov exponents



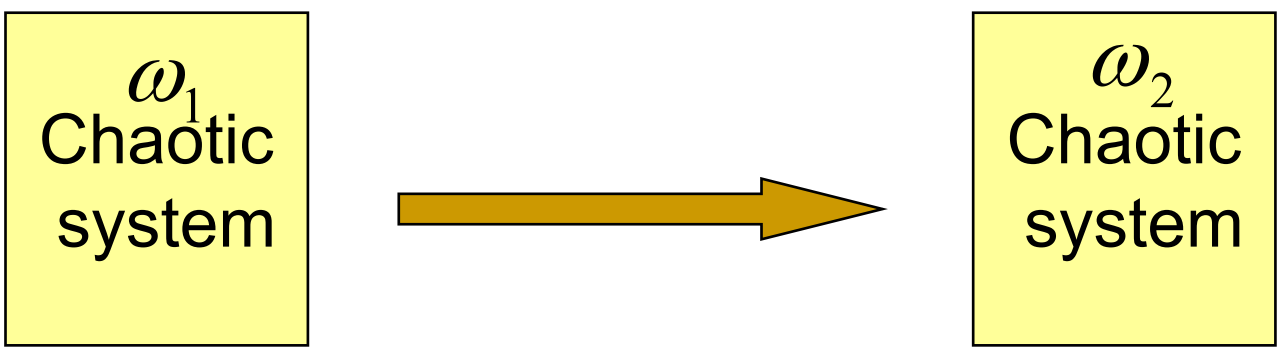
Optimal prediction time = the characteristic period

Phase synchronization



M. Rosenblum, A. Pikovsky, J. Kurths, PRL **76**, 1804 (1996)
M. Rosenblum, A. Pikovsky, J. Kurths, PRL **78**, 4193 (1997)

How does the phase synchronization manifest itself in unidirectionally coupled chaotic systems?



Simple model

Phase lead synchronization (PLS)

$$\dot{\varphi}_1 = \omega_1,$$
$$\dot{\varphi}_2 = \omega_2 + \varepsilon \sin(\varphi_1 - \varphi_2)$$

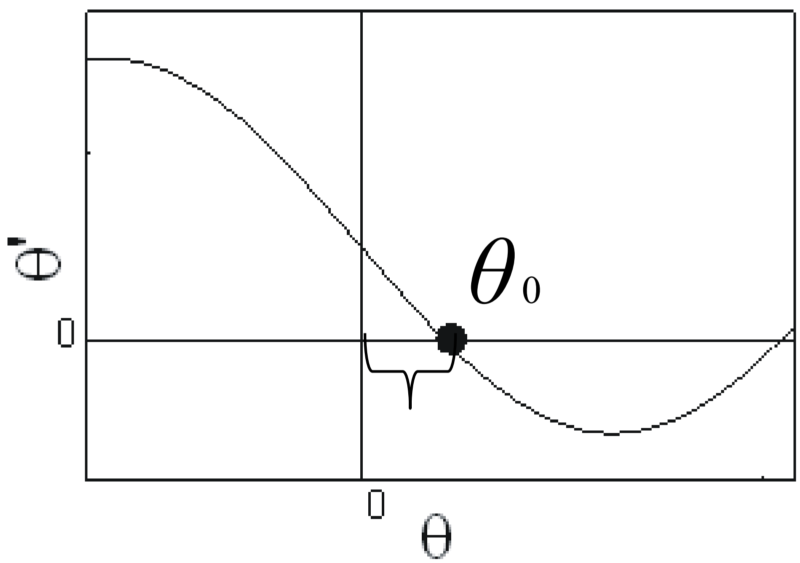
$$\omega_2 > \omega_1$$

Adler equation

$$\dot{\theta} = \omega_2 - \omega_1 - \varepsilon \sin\theta$$

$$\theta = \varphi_2 - \varphi_1$$

Phase lead synchronization (PLS)



$\omega_2 > \omega_1$

$$\varphi_2 = \varphi_1 + \theta_0 \qquad \theta_0 = \arcsin((\omega_2 - \omega_1)/\varepsilon) > 0$$

Is it possible to obtain the phase lead synchronization in unidirectionally coupled chaotic oscillators?

Two chaotic Rössler systems

Drive system

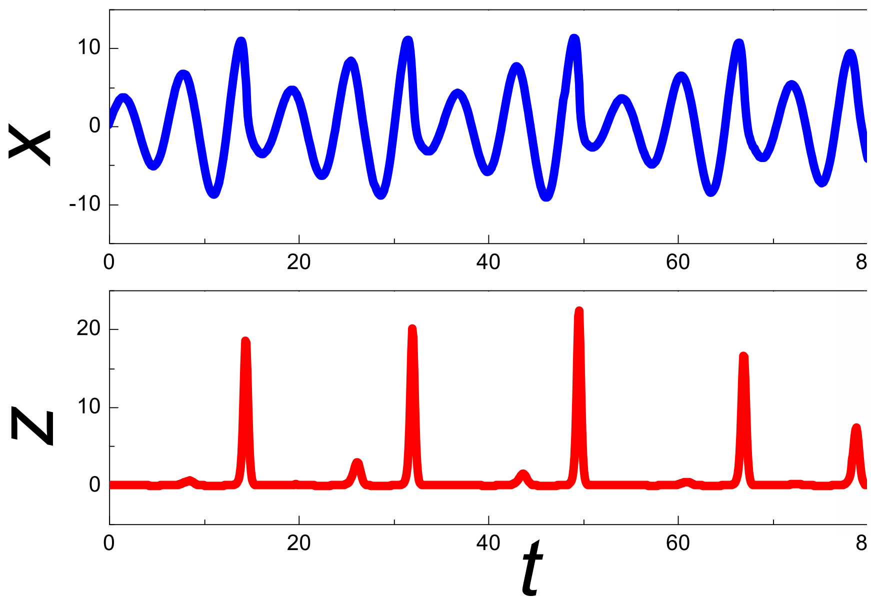
$$\dot{x}_1 = -\omega_1 y_1 - z_1,$$
$$\dot{y}_1 = \omega_1 x_1 + a y_1,$$
$$\dot{z}_1 = b + z_1(x_1 - c).$$

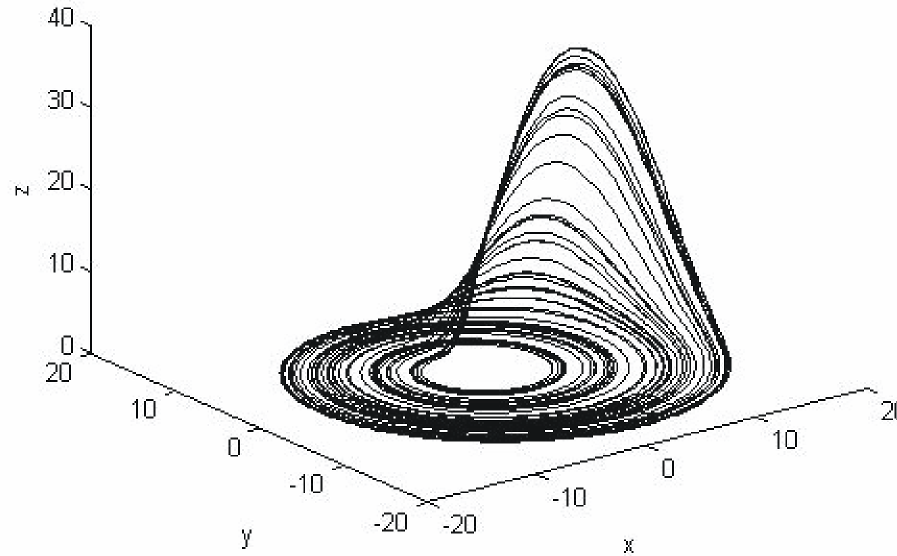
Response system

$$\dot{x}_2 = -\omega_2 y_2 - z_2 + \boxed{\varepsilon(x_1 - x_2)},$$
$$\dot{y}_2 = \omega_2 x_2 + a y_2,$$
$$\dot{z}_2 = b + z_2(x_2 - c).$$

$$a = 0.165, b = 0.2, c = 10, \omega_1 = 0.95, \omega_2 = 0.99$$

Phase notion for chaotic system



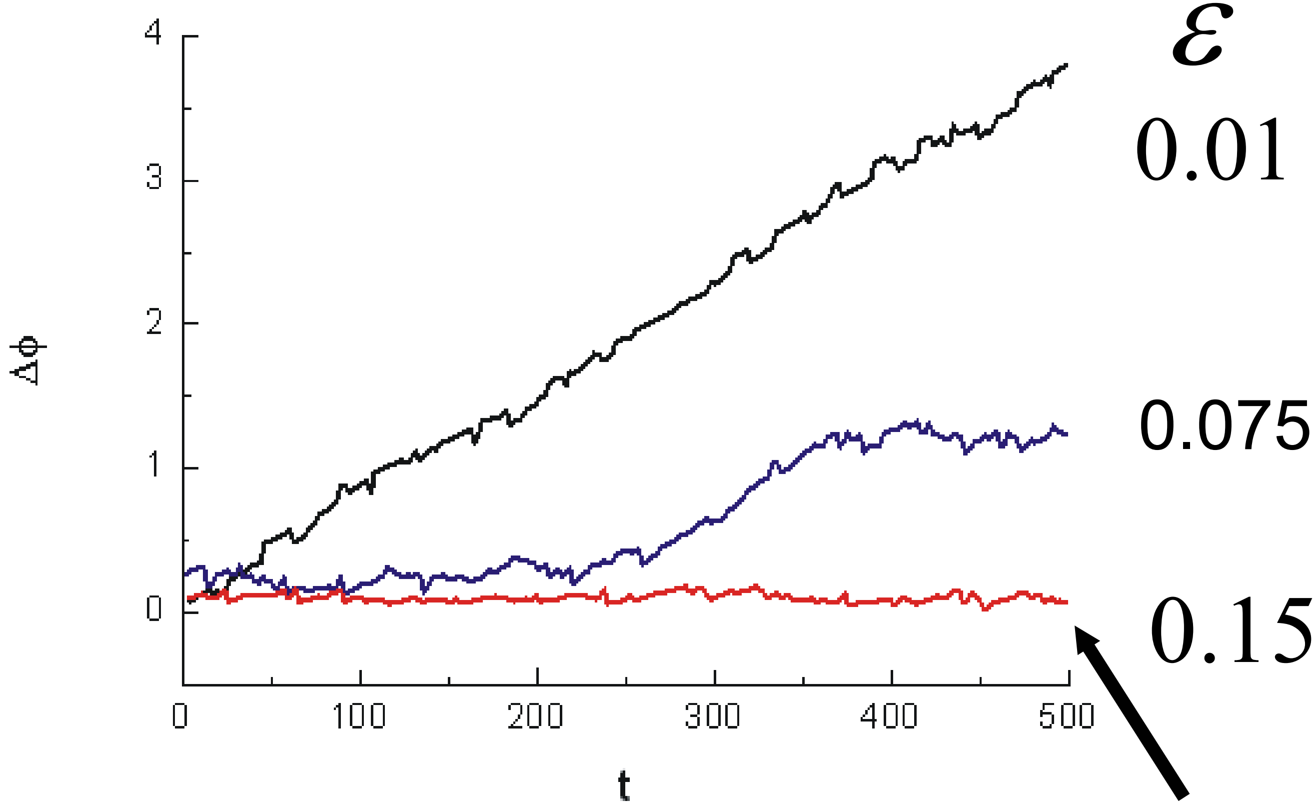


$$\varphi_{1,2} = \arctan \frac{y_{1,2}}{x_{1,2}}, \; A_{1,2} = (x_{1,2}^2 + y_{1,2}^2)^{1/2}.$$

Almost all the time the Rössler system is situated on the plane (x,y) moving along the unstable spiral

Phase dynamics

Coupling strength

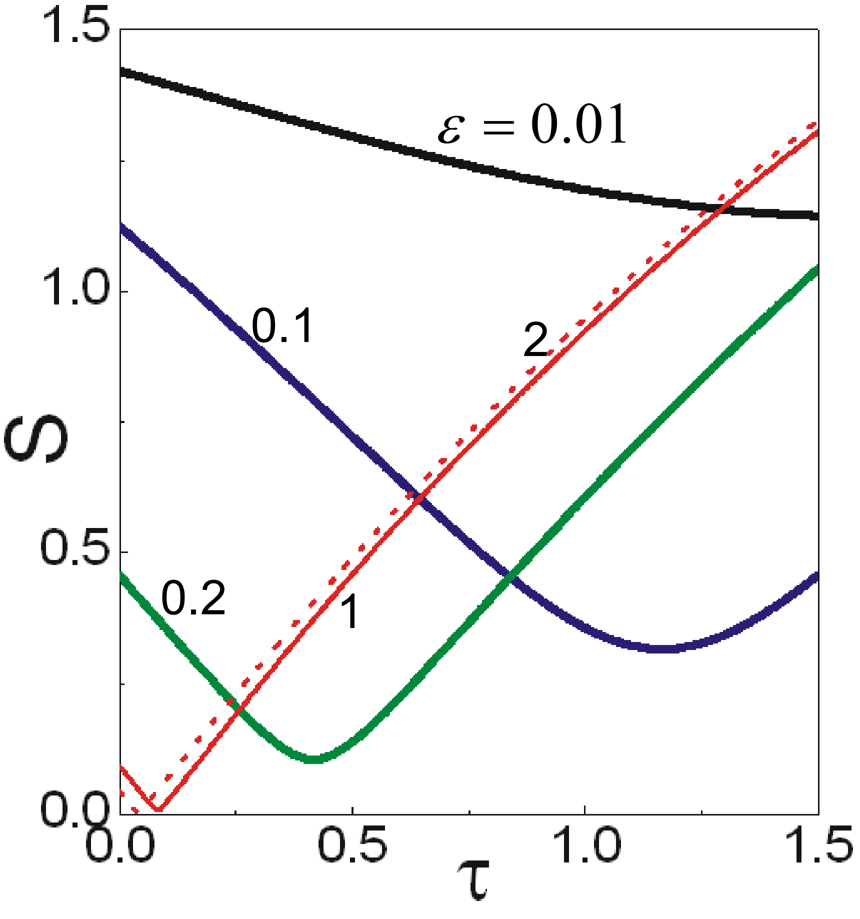


$$\Delta\varphi = \varphi_2 - \varphi_1 > 0$$

Phases are locked

We have the phase lead synchronization

Anticipating synchronization?



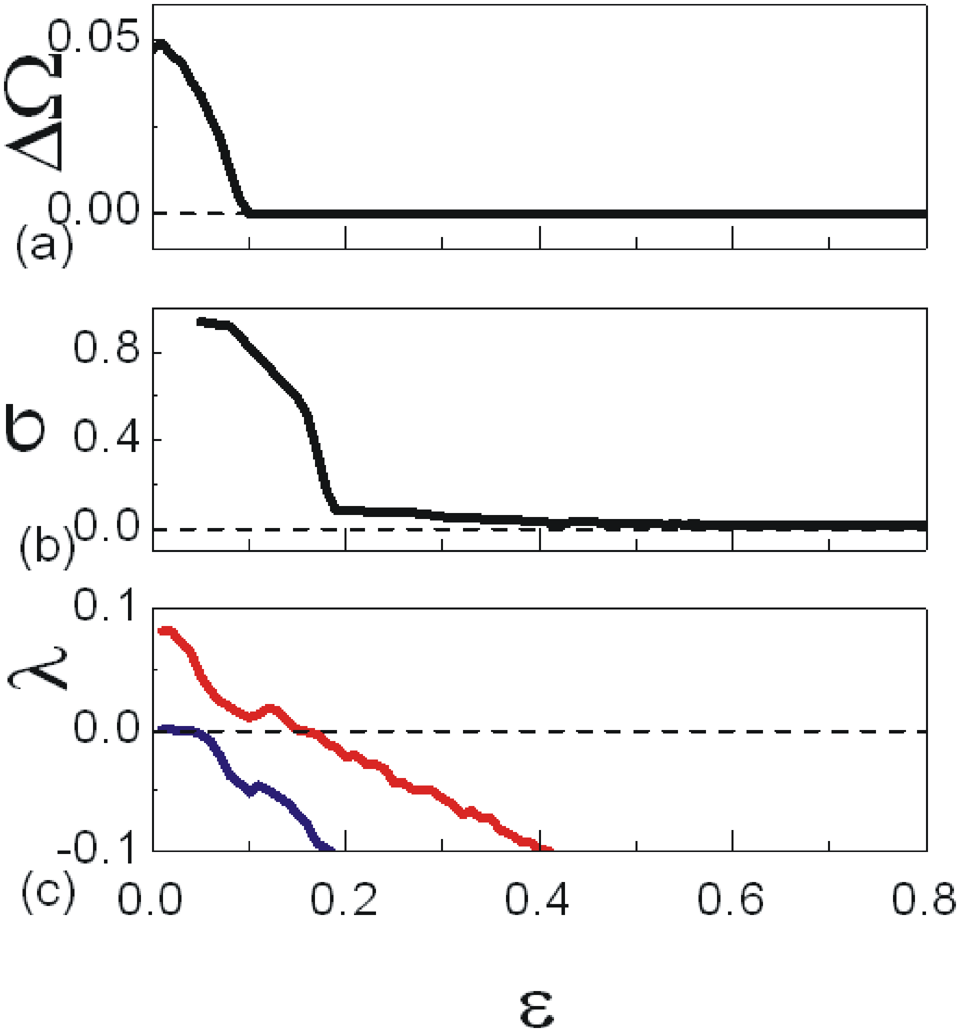
$$S^2(\tau) = \frac{<[x_2(t) - x_1(t+\tau)]^2>}{[<x_1^2><x_2^2>]^{1/2}}$$

Phase lead synchronization - the similarly function has a deep minimum

The further increase of the coupling strength - anticipating synchronization:

$$x_2(t) = x_1(t+\tau)$$

Transitions



$$\Omega_i = <\dot{\varphi}>, \Delta\Omega = \Omega_2 - \Omega_1$$

$$\sigma = \min S(\tau)$$

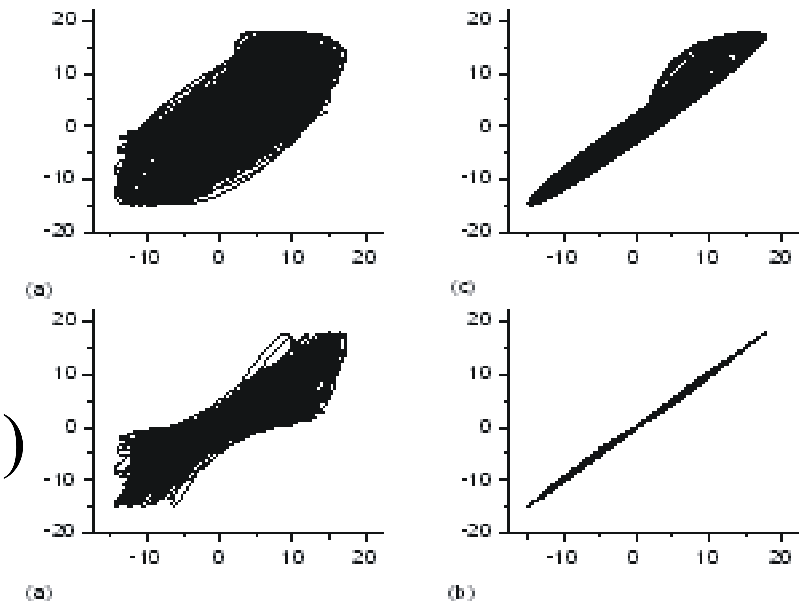
Two largest conditional Lyapunov exponents

Anticipating synchronization

$\varepsilon = 0.15$ $\varepsilon = 0.4$

$x_1(t)$

$x_1(t+\tau)$

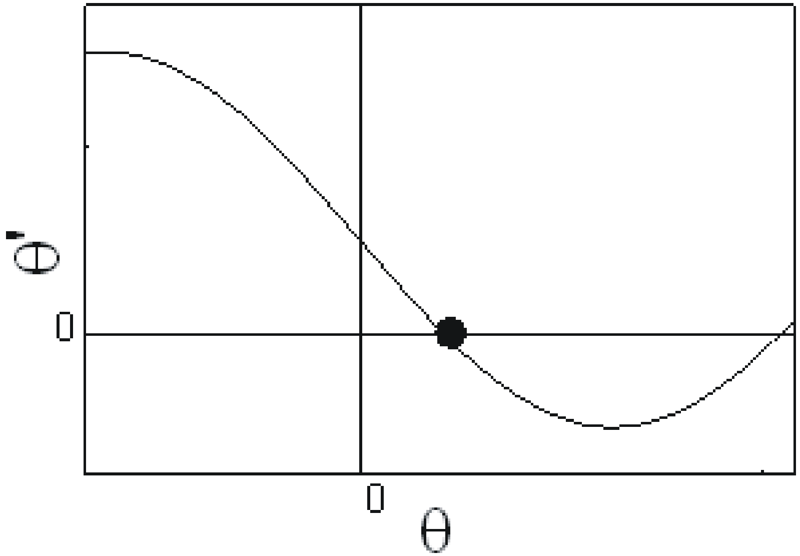


$x_2(t)$

Analytical treatment

Asymptotical Adler equation

$$\dot{\theta} = \omega_2 - \omega_1 - \frac{\varepsilon}{2}\frac{A_1}{A_2}\sin\theta$$



Positive stable fixed point

$$\theta = \varphi_2 - \varphi_1$$

$$\theta_0 = \arcsin\left[\frac{2A_2(\omega_2 - \omega_1)}{A_1\varepsilon}\right] > 0,$$

$$\varepsilon > 0.08$$

$$\varepsilon > 2A_2(\omega_2 - \omega_1)/A_1$$

$$\omega_1 = 0.95, \omega_2 = 0.99$$

Conclusions

- A new phenomenon of phase lead synchronization has been detected and analyzed numerically and analytically in unidirectionally coupled chaotic Roessler systems.
- The phase lead synchronization transforms to the anticipating synchronization for sufficiently strong coupling strength.
- The anticipating of the dynamics can appear in unidirectionally coupled chaotic systems without delay terms in either drive or response systems provided the response system is faster then the drive.

Thank you

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