Adaptive delayed feedback control algorithm with a state dependent delay

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Outline

- Delayed feedback control method
- Overview of publications
- Problem formulation
- The idea of algorithm
- Demonstration for Roessler system
- Demonstration for Mackey Glass system
- Conclusions

Delayed feedback control (DFC) method

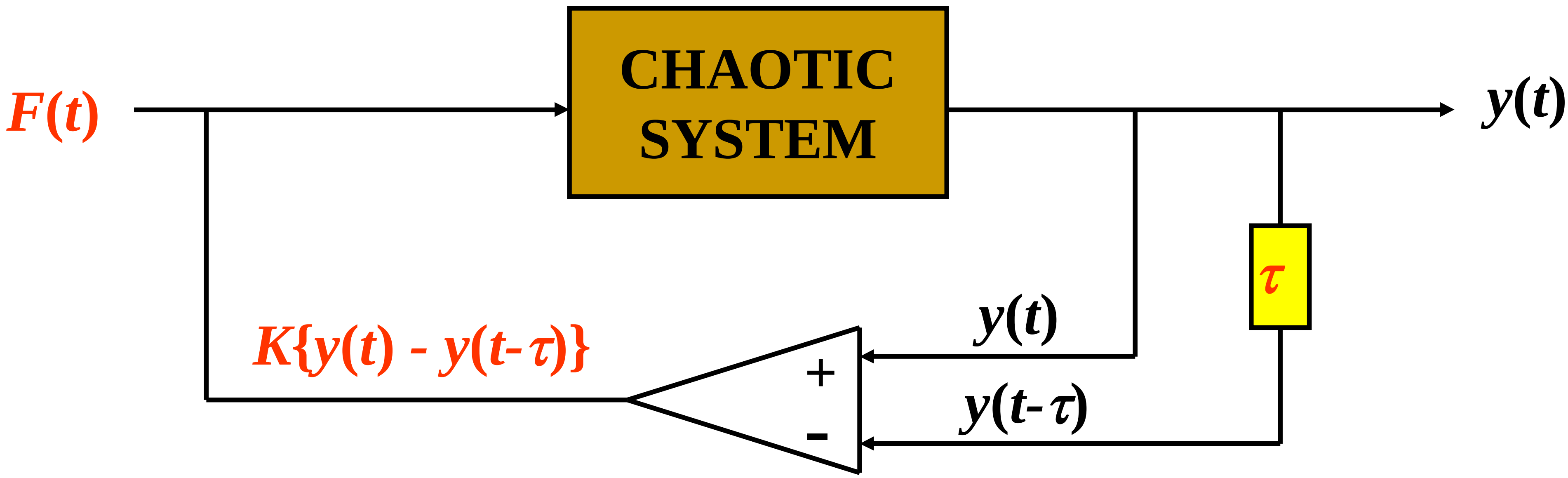
K. Pyragas, Phys. Lett. A **170**, 421 (1992)

$$\frac{dy}{dt} = P(\vec{x}, y) + F(t)$$

$$\frac{d\vec{x}}{dt} = \vec{Q}(\vec{x}, y)$$

$F(t) = K\{y(t) - y(t-\tau)\}$

$\tau = T$ - *the period of unstable orbit*



Overview of publications (I)

Reviews:

E. Schoell and **H. G. Schuster**, Handbook of Chaos Control (2008);

K. Pyragas, in Phylos. Trans. R. Soc. London, Ser. A **364**, 2309 (2006);

Overview of publications (II)

DFC with state dependent delays:

W. Michiels, V. Van Assche, and **S. Niculescu,**
IEEE Trans. Autom. Control, **50** (4), 493 (2005);

A. Gjurchinovski and **V. Urumov,** EPL **84**, 40013 (2008);
PRE **81**, 016209 (2010);

Variation of delay enhances the stability of the DFC algorithm

Overview of publications (III)

The periods of UPOs are not known a priori

Adaptation of delay via discrete iterative algorithms:

A. Kittel, J. Parisi, and **K. Pyragas**, PLA **198**, 433 (1995);
G. Chen, **X. Yu**, IEEE Trans. Sirc. Syst., **46** (6), 767(1999);
X. Yu, IEEE Trans. Sirc. Syst., **46** (11), 1408 (1999);

Theoretical background:

W. Just, D. Reckwerth, J. Moeckel, H. Benner, PRL. **81**, 562 (1998);

Continuous-time methods:

G. Herrmann, PLA 2**87**, 245 (2001);
W. Lin, H. Ma, J. Feng, G. Chen, PRE **82**, 046214 (2010);

Problem formulation

Our aim is:

To construct a continuous-time adaptive DFC scheme with the state-dependent time delay for an automatic search for the periods of the target UPOs

We use the idea of continuous-time gradient-descent method developed by:

F. Sorrentino and **E. Ott,** PRL **100**, 114101 (2010);

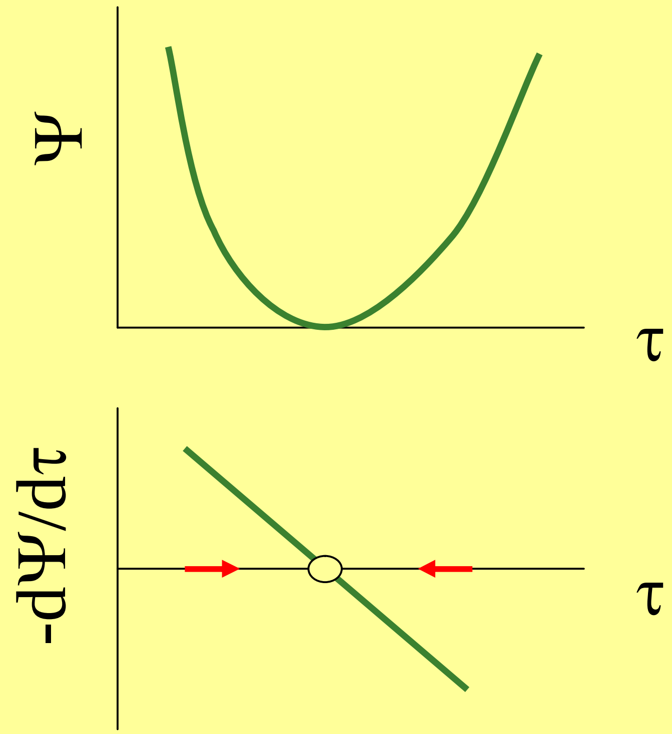
The idea of the algorithm

Gradient descent method:

$$\dot{X}(t) = F[X(t), K\Delta s(t)];$$

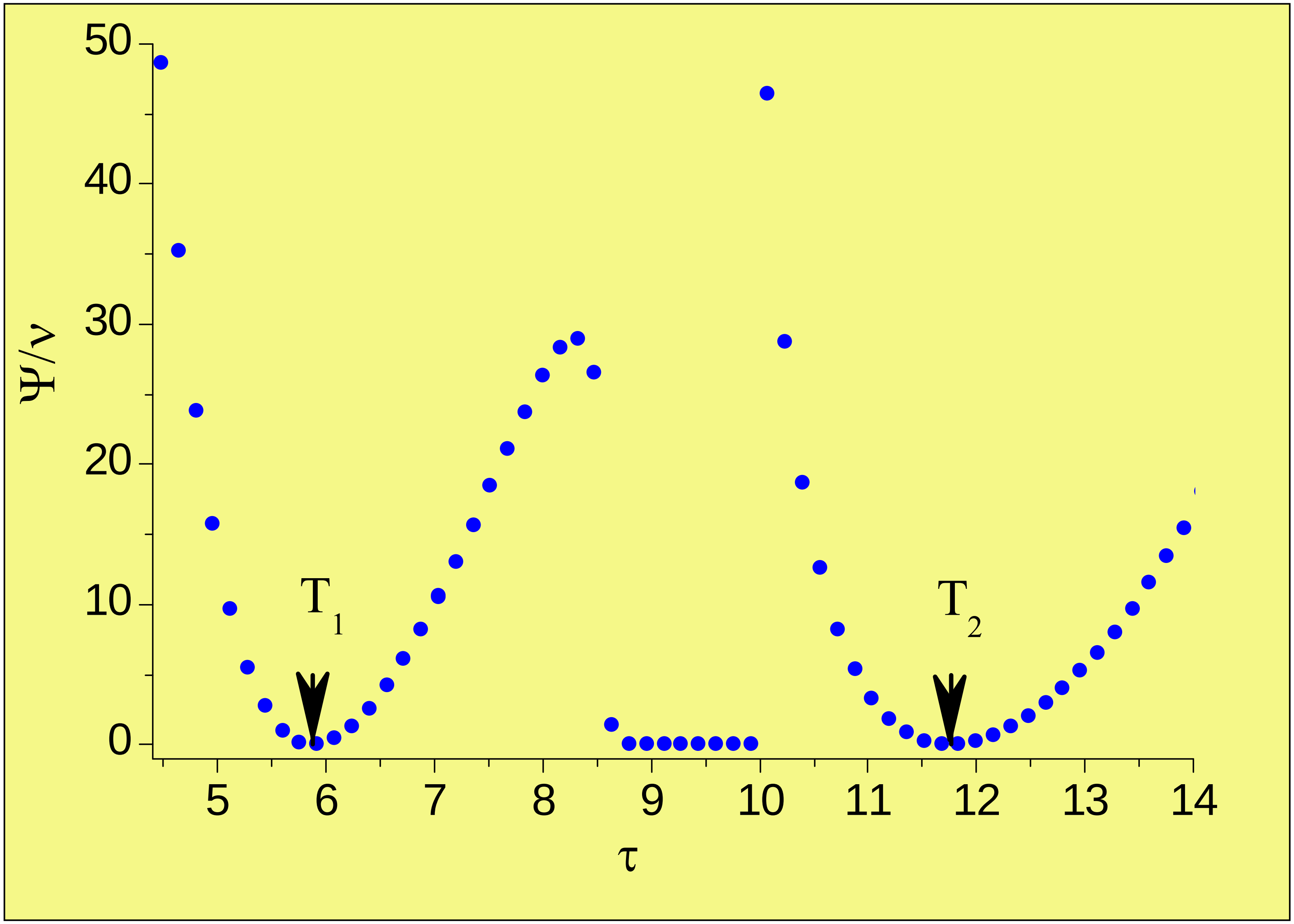
$$\Delta s(t) = g[X(t)] - g[X(t-\tau)];$$

$$\Psi = \int_0^t dt' e^{-\nu(t-t')}[s(t') - s(t'-\tau(t'))]^2$$



$$\frac{d\tau}{dt} = -\beta \frac{d\Psi}{d\tau}$$

The functional for the Roessler system



The full system for adaptive search of delay

$$\dot{X}(t) = F(X(t), K[s(t) - s(t-\tau(t))]),$$
$$\dot{u} = \gamma[s(t-\tau(t)) - u],$$
$$\dot{G} = -\nu G + 2\gamma[s(t) - s(t-\tau(t))][s(t-\tau(t)) - u],$$
$$\dot{\tau} = -\beta G.$$

$$G(t) \equiv \frac{d\Psi}{d\tau}$$

$$T << 1/\nu << 1/\beta,$$
$$1/\gamma << T.$$

The integral expressions are excluded !

Demonstration for the Roessler system

$$\dot{x} = -\omega y - z,$$

$$\dot{y} = \omega x + ay - K[y(t) - y(t - \tau(t))],$$

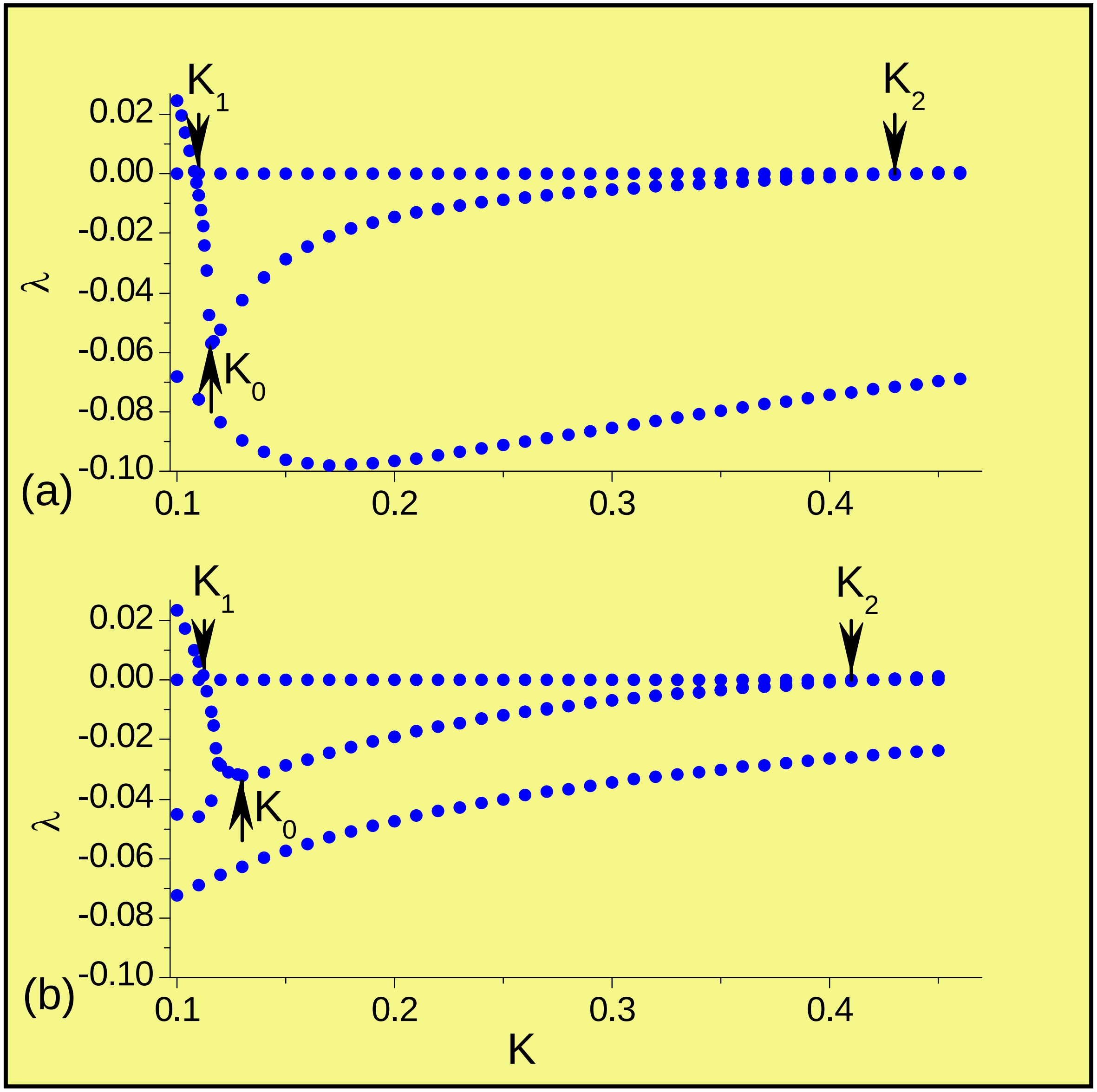
$$\dot{z} = b + z(x - c),$$

$$\dot{u} = \gamma[y(t - \tau(t)) - u],$$

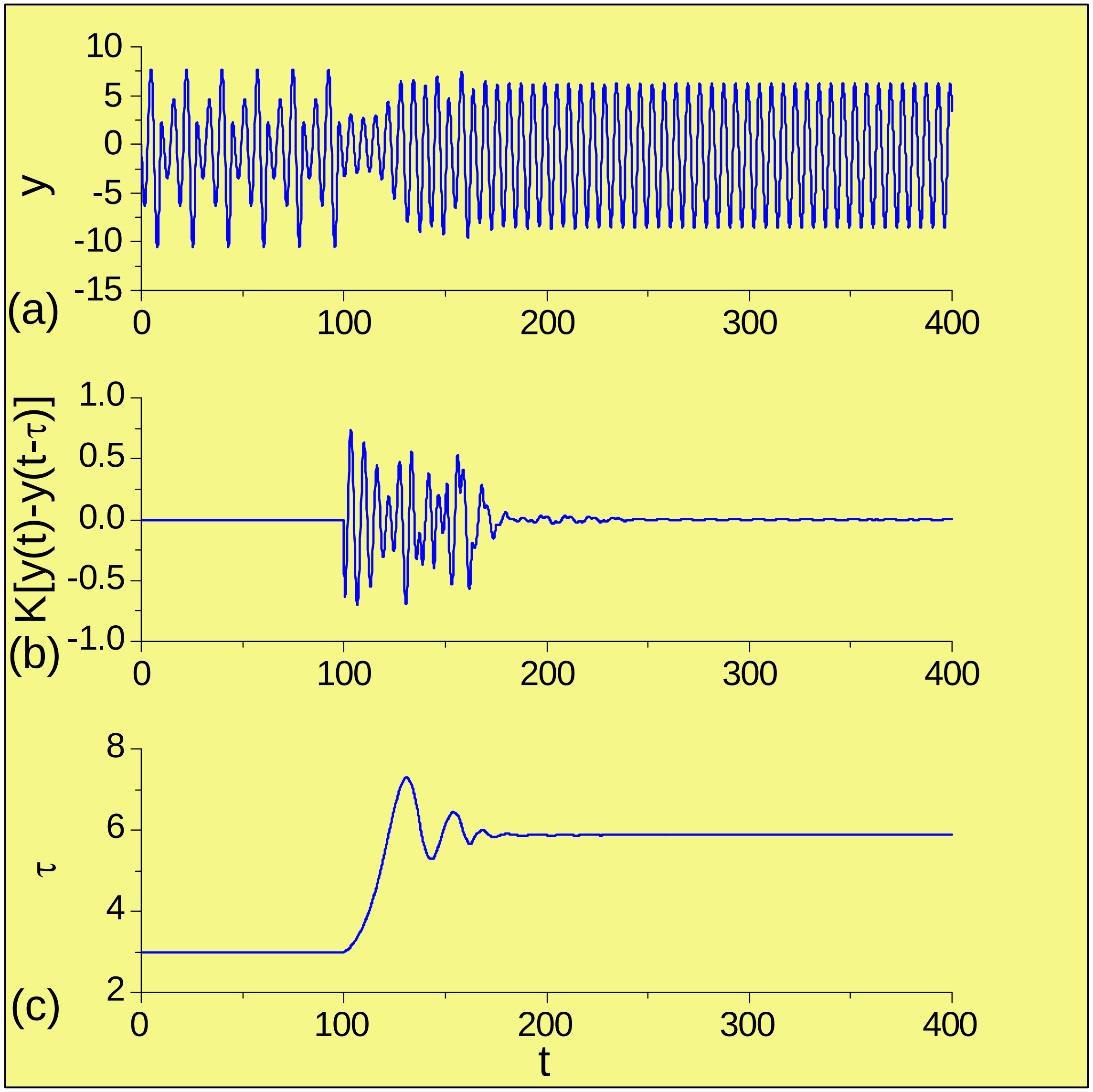
$$\dot{G} = -\nu G + 2\gamma[y(t) - y(t - \tau(t))][y(t - \tau(t)) - u],$$

$$\dot{\tau} = -\beta G.$$

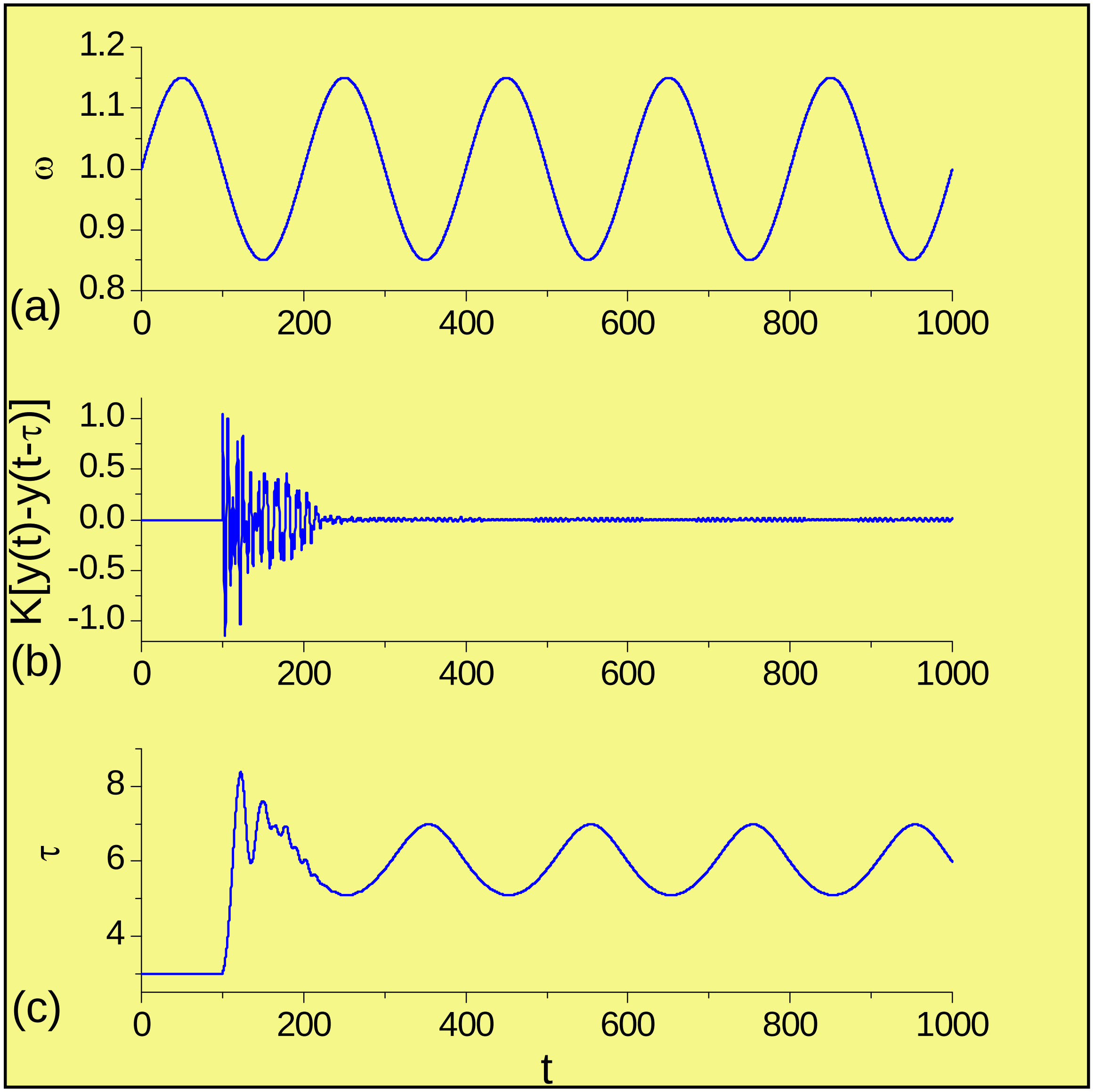
Lyapunov exponents of the 1-st and 2-nd UPOs



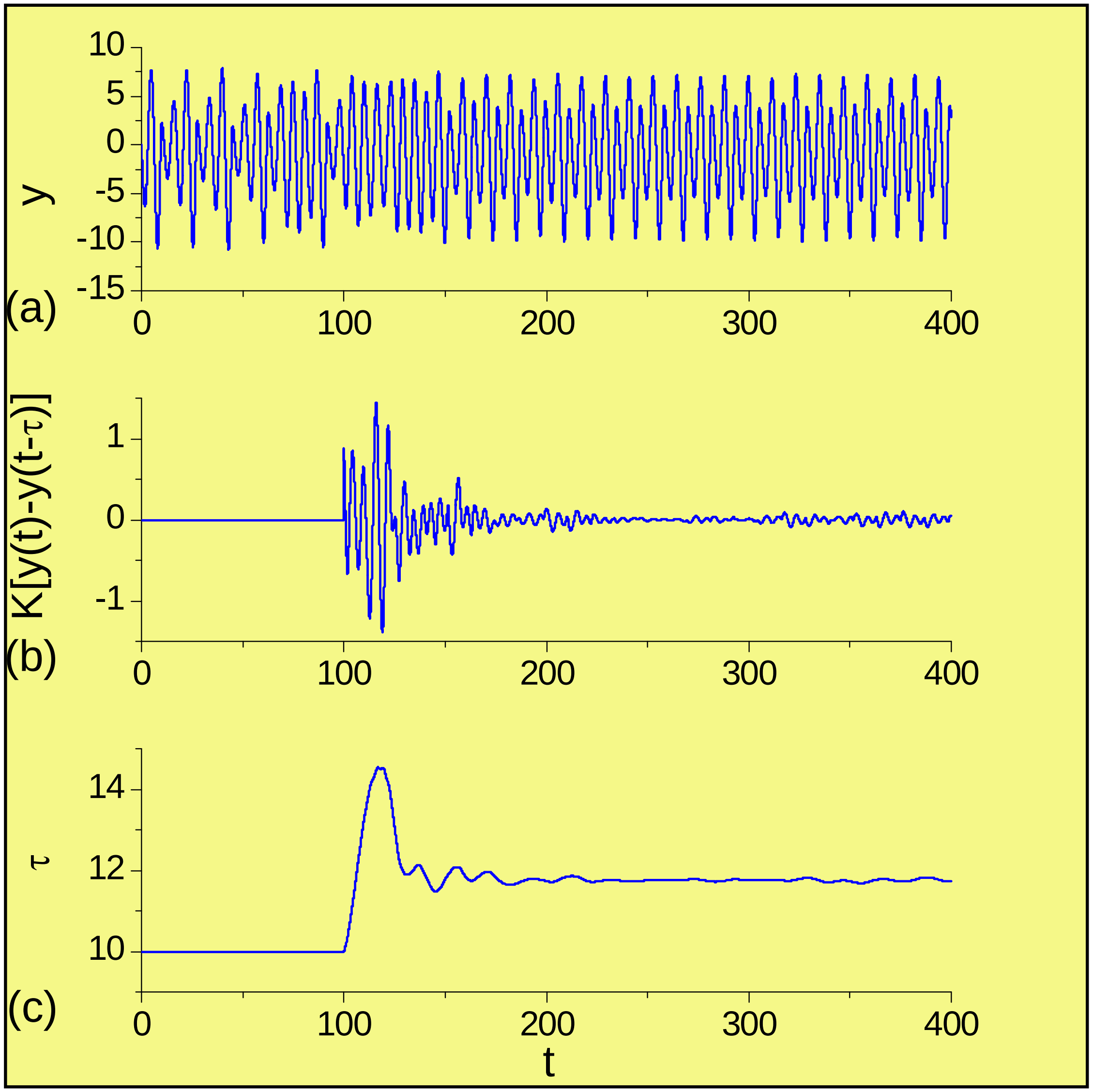
Adaptive stabilization of the 1-st UPO



Periodic variation of the frequency parameter



The period-2 UPO in the presence of noise



Demonstration for the Mackey Glass system

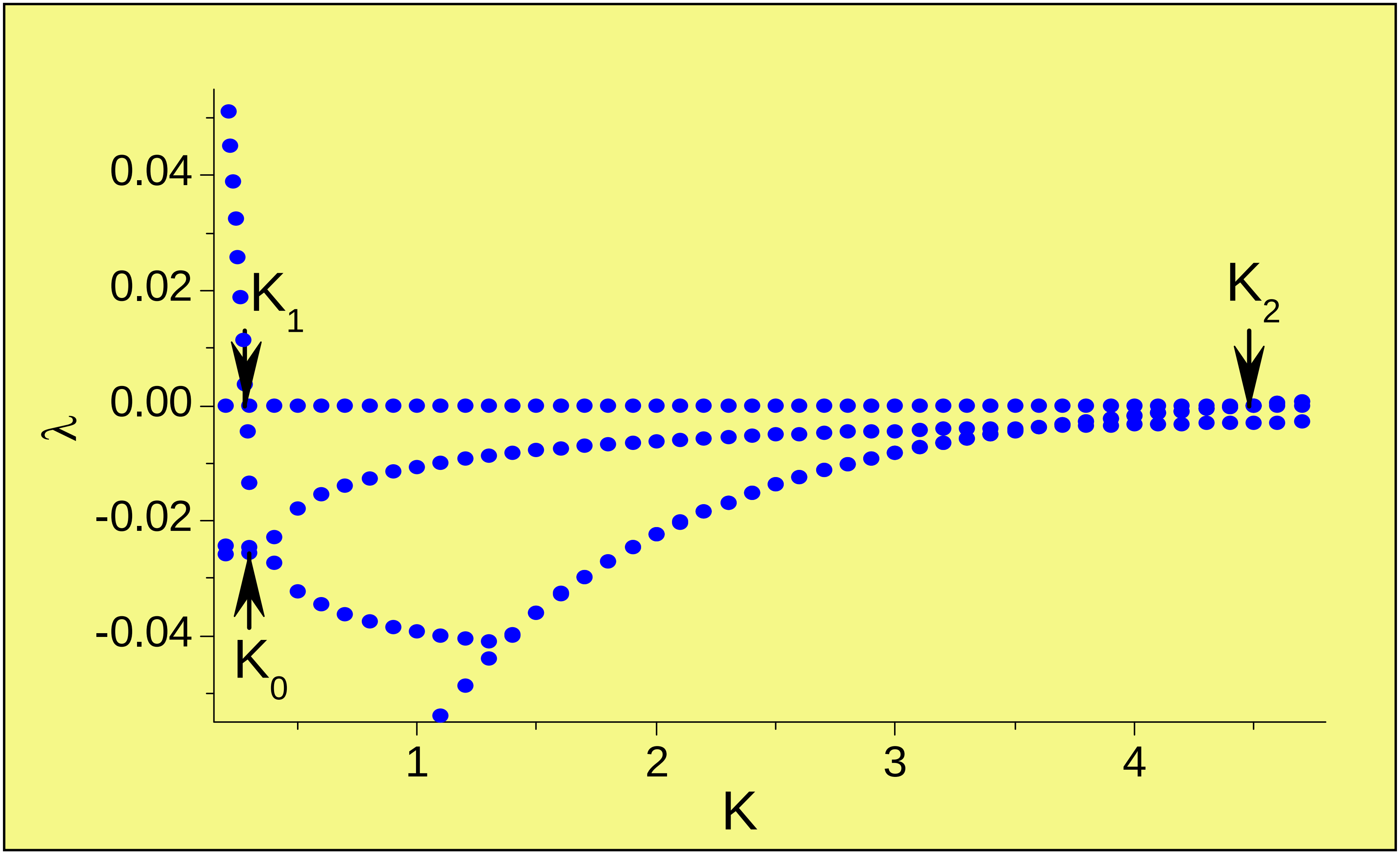
$$\dot{x} = \frac{ax(t-\tau_s)}{1+x^b(t-\tau_s)} - cx - K[x(t) - x(t-\tau(t))],$$

$$\dot{u} = \gamma[x(t-\tau(t)) - u],$$

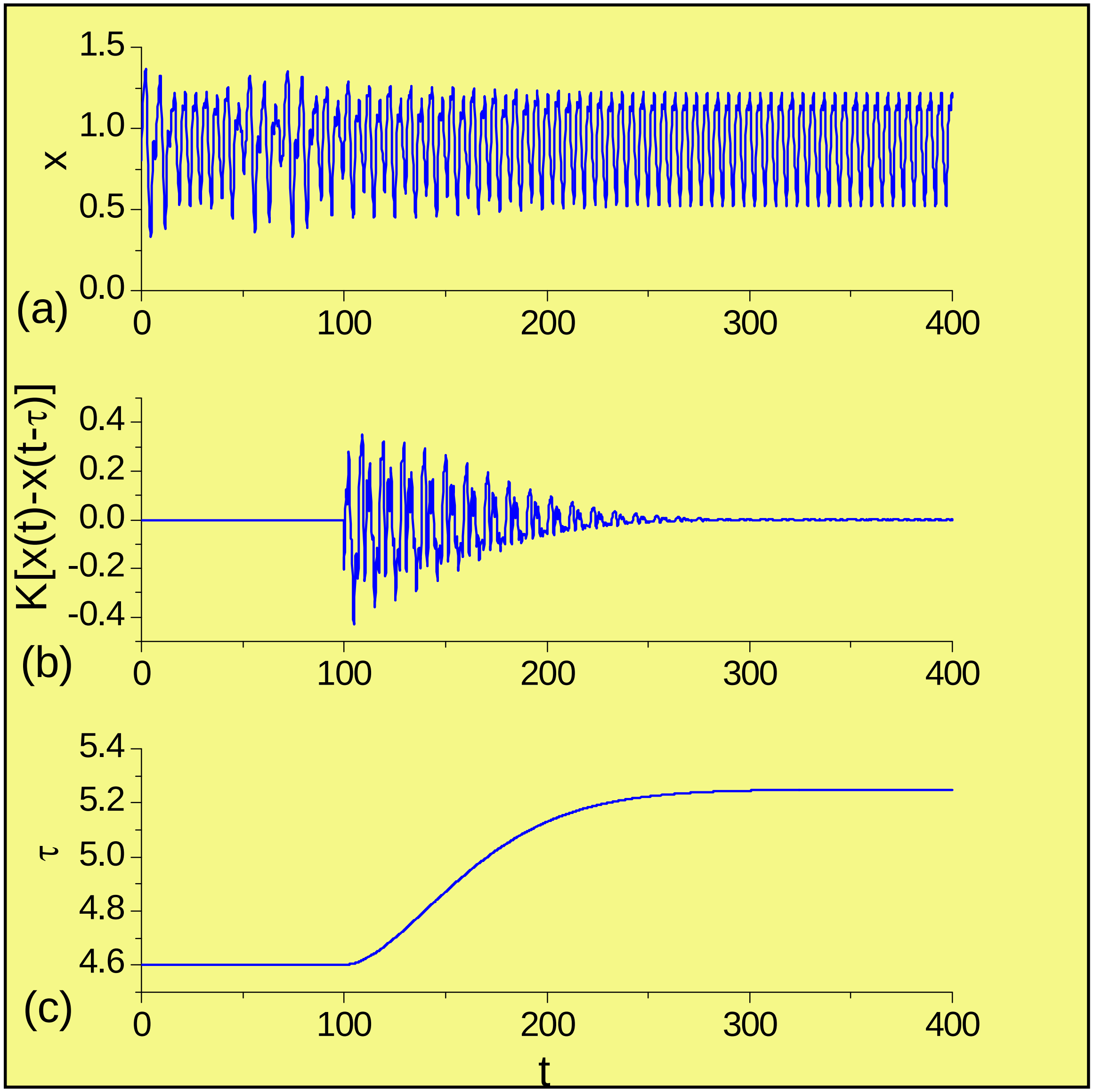
$$\dot{G} = -\nu G + 2\gamma[x(t) - x(t-\tau(t))][x(t-\tau(t)) - u],$$

$$\dot{\tau} = -\beta G.$$

Lyapunov exponents for the 1-st UPO



Adaptive stabilization of the 1-st UPO



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Conclusions

- We have proposed a continuous-time adaptive DFC algorithm that uses the gradient descent method for the search of the unknown UPO period.

- The state-dependent delay converges towards the period of desired orbit even if the delay time starts relatively far from the actual period.

- The algorithm enables to find the UPOs of DDE systems without using the Puancare section (for DDE systems such sections are rather problematic).

- The algorithm may be used for stabilization and tracking of experimental systems when the dynamic equations are unknown.

Acknowledgments

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