Continuous pole placement for delayed feedback controlled systems

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Outline

- Introduction
- Problem formulation
- The idea of algorithm
- Demonstration for the Lorenz system
- Demonstration for the Roessler system
- Conclusions

Introduction

DFC algorithm:

K. Pyragas, Phys. Lett. A **170**, 421 (1992)

Reviews:

K. Pyragas, Phylos. Trans. R. Soc. London, Ser. A **364**, 2309 (2006);

E. Schoell and **H. G. Schuster**, Handbook of Chaos Control (2008);

P. Hoevel, Control of Complex Nonlinear Systems with Delay (2010) ;

Problem formulation

TDFC algorithm is successful if the real parts of all nontrivial FEs are negative. Suppose that for a given input matrix we know some particular control matrix (this knowledge may be based, e.g. on an empirical approach) that stabilizes the desired UPO. Our aim is to optimize the components of the control matrix in such a way as to achieve the fastest approach to the stabilized orbit, i.e. to make the minimum of the leading FE as deep as possible.

This problem is nontrivial, since the number of FEs is infinite while the number of the control parameters in the control matrix is finite. For this aim we adapt the pole placement method with time-delay to the case of periodic time-delay systems.

Our aim is to induce some small desired changes into real parts of several leading FEs by applying small carefully estimated perturbations to the components of the control vector. Such a manipulation can be accomplished by using the matrix of sensitivity.

$$s_{ij} \equiv \frac{\partial \lambda_i}{\partial k_j}$$

$$\Delta\Lambda = S\Delta k \quad \Rightarrow \quad \Delta k = S^{+}\Delta\Lambda$$

The idea of the algorithm (I)

Stabilizing an UPO with TDFC method:

$$\dot{X}(t) = F[X(t)] - \kappa(bk^T)[X(t) - X(t-\tau)]$$

$$\dot{\xi}(t) = F[\xi(t)]$$

$$\xi(t) = \xi(t+\tau)$$

The idea of the algorithm (II)

Algorithm of continuous pole placement method:

1. For a given input vector b take the particular initial control vector k obtained by some empirical approach, which stabilizes the controlled orbit.
2. Scan the dependence of several FEs with the largest real parts on the feedback gain $\varkappa$. Find the minimal value Re λ_{min} of the real part of the leading FE and the corresponding feedback gain $\varkappa_{min}$.
3. By trial and error, select for some $\varkappa_1$ (close to $\varkappa_{min}$) a set of FEs whose real parts should be shifted/held towards more negative values and compute the matrix of sensitivity.
4. Shift/hold the real parts of the chosen FEs and perform simultaneously the Newton-Broyden iterations in order to minimize the error of the shifts. The control parameters are changed iteratively. When the Newton-Broyden method does not yield satisfactory corrections, stop shifting and go to step 2. Repeat Steps 2-4 until the iterations of Re λ_{min} do not reach minimum. The vector obtained at the last iteration is an optimal vector.

The idea of the algorithm (III)

Floquet decomposition for TDFC systems

$$\dot{X}(t) = F[X(t)] - \kappa \mathbf{K}[X(t) - X(t-\tau)]$$

$$\delta X(t) = X(t) - \xi(t)$$

$$\delta \dot{X}(t) = DF[\xi(t)]\delta X(t) - \kappa \mathbf{K}[\delta X(t) - \delta X(t-\tau)]$$

$$\delta X(t) = u(t)e^{\lambda t}$$
$$u(t) = u(t+\tau)$$

$$\dot{u}_i(t) = [\mathbf{A}(t) - \lambda_i \mathbf{I}]u_i(t) - \kappa \mathbf{K} u_i(t)[1 - e^{-\lambda_i \tau}]$$

The idea of the algorithm (IV)

Reformulating the problem in the form of operators:

$$\hat{L}_i u_i(t) = 0$$

$$\hat{L}_i = \frac{d}{dt} - \mathbf{A}(t) + \lambda_i \mathbf{I} + \kappa \mathbf{K}[1 - e^{-\lambda_i \tau}]$$

$$\langle v | u \rangle = \int_0^\tau dt\, v^+(t) u(t)$$

$$\langle L^+ v | u \rangle = \langle v | L u \rangle$$

$$\hat{L}_i^+ = -\frac{d}{dt} - \mathbf{A}^T(t) + \lambda_i^* \mathbf{I} + \kappa \mathbf{K}[1 - e^{-\lambda_i^* \tau}]$$

$$\hat{L}_i^+ v_i(t) = 0$$

$$u_i(0) = u_i(\tau)$$
$$v_i(0) = v_i(\tau)$$

The idea of the algorithm (V)

Evaluating the matrix of sensitivity for TDFC systems

$$\mathbf{K} = bk^T$$

$$\frac{\partial}{\partial k_j}(\hat{L}_i u_i(t)) = \frac{\partial L_i}{\partial k_j} u_i(t) + L_i \frac{\partial u_i}{\partial k_j} = 0$$

$$\left\langle v_i \left| \frac{\partial L_i}{\partial k_j} u_i(t) \right. \right\rangle = 0$$

$$s_{ij} \equiv \frac{\partial \lambda_i}{\partial k_j} = (\mu_i^{-1} - 1) \frac{\left\langle v_i \middle| \kappa(b e_j^T) \middle| u_i \right\rangle}{\left\langle v_i \middle| I + \tau \mu_i^{-1} \kappa(b k^T) \middle| u_i \right\rangle}$$

The idea of the algorithm (VI)

Alternative approach:

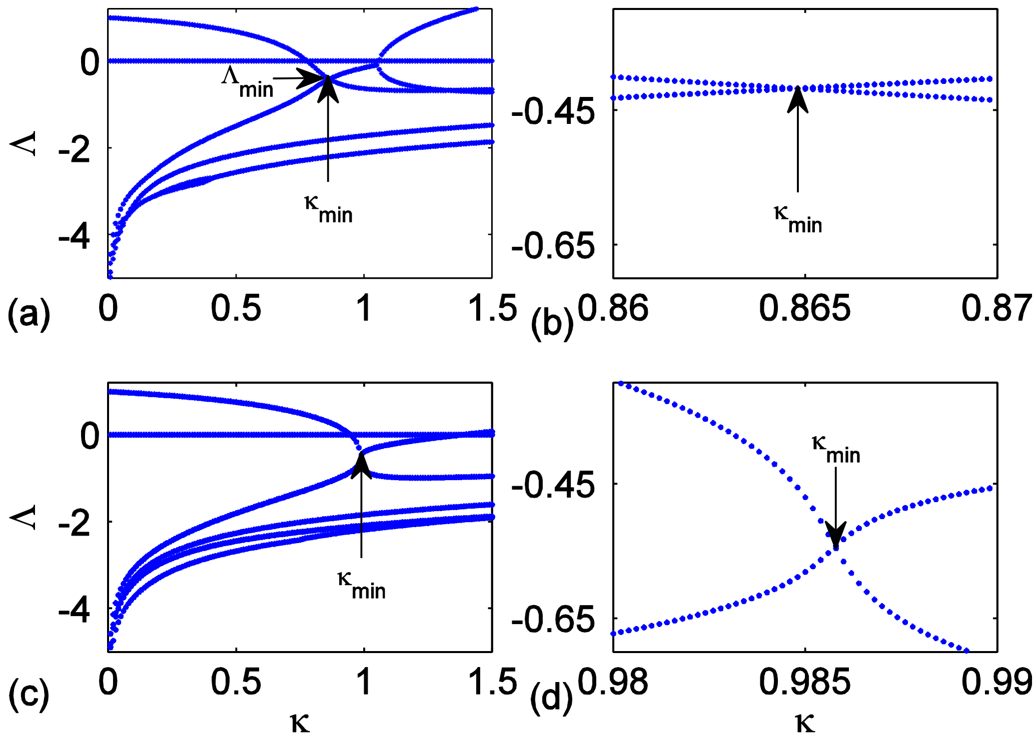
Evaluating the matrix by finite differences

$$S_{ij} = \frac{\Lambda_i(k + \varepsilon e_j) - \Lambda_i(k - \varepsilon e_j)}{2\varepsilon}$$

Demonstration for the Lorenz system (I)

The results:

Global and zoomed view at the start and finish



Demonstration for the Lorenz system (II)

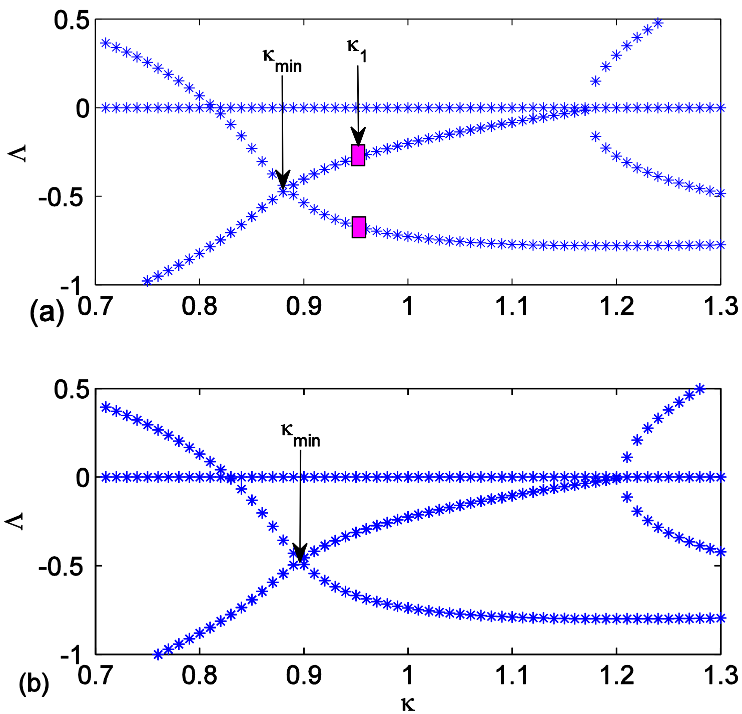
Table of results:

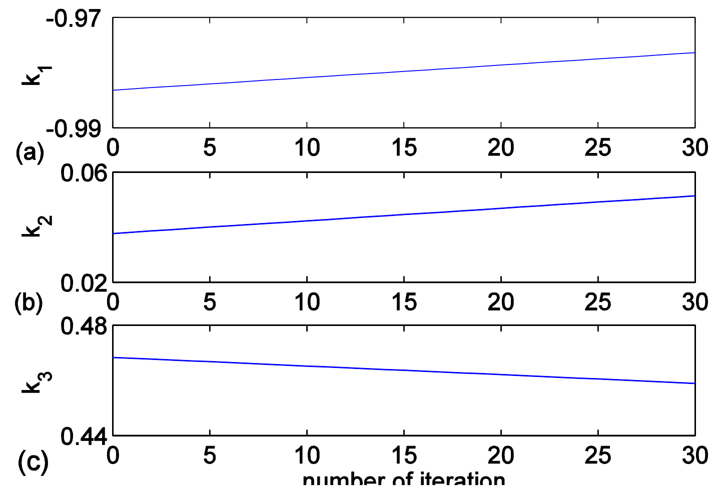
Table 1. Shifting the FEs in the Lorenz system

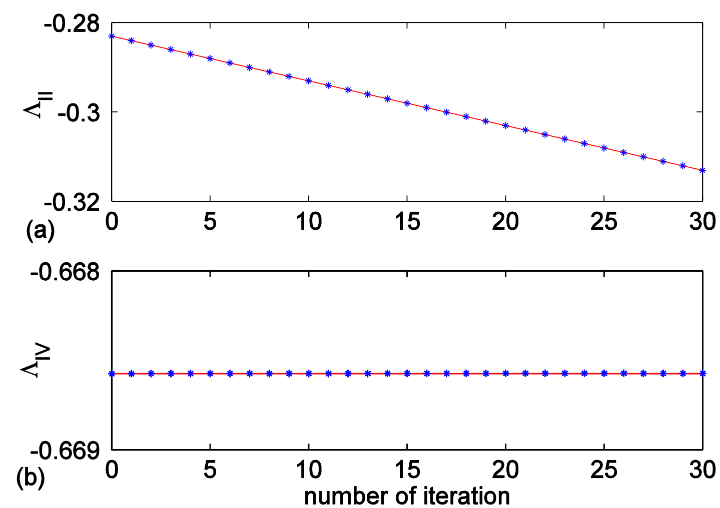
No.	$\mathbf{k}^T$	κ_1	FEs	$\Delta\mathbf{\Lambda}^T/10^{-3}$	N_{it}
1	$[-1, 0, 0.5]$	0.95	$[II, IV]$	$[-1, 0]$	10
2	$[-0.99773, 0.00459, 0.49680]$	0.95	$[II, IV]$	$[0, -1]$	10
3	$[-0.99709, 0.00651, 0.49456]$	0.95	$[II, IV]$	$[-1, -1]$	39
4	$[-0.98537, 0.03225, 0.47374]$	0.95	$[II, IV]$	$[0, -1]$	25
5	$[-0.98319, 0.03774, 0.46836]$	0.95	$[II, IV]$	$[-1, 0]$	30
6	$[-0.97639, 0.05141, 0.45899]$	0.95	$[II, IV]$	$[-1, 0]$	30
7	$[-0.96965, 0.06509, 0.44943]$	0.95	$[II, IV]$	$[-1, 0]$	30
8	$[-0.96288, 0.07896, 0.43955]$	0.97	$[II, IV]$	$[-1, 0]$	40
9	$[-0.95418, 0.09791, 0.42708]$	0.97	$[II, IV]$	$[-1, 0]$	20
10	$[-0.94973, 0.10772, 0.42049]$	1.0	$[II, IV]$	$[-1, 0]$	40
11	$[-0.94060, 0.12674, 0.40851]$	1.0	$[II, IV]$	$[-1, 0]$	45

Demonstration for the Lorenz system (III)

Example of shifting roots (No. 5) :







Demonstration for the Roessler system (I)

Extending the algorithm to the ETDFC systems

$$\dot{X}(t) = F[X(t)] - \kappa(bk^T)[X(t) - (1-R)B(t-\tau)]$$

$$B(t) = X(t) + RB(t-\tau)$$

$$\dot{u}_i(t) = [\mathbf{A}(t) - \lambda_i \mathbf{I}]u_i(t) - \kappa(bk^T)u_i(t)\frac{1-e^{-\lambda_i \tau}}{1-e^{-\lambda_i \tau}R}$$

$$s_{ij}^k = \frac{\partial \lambda_i}{\partial k_j}$$

$$s_i^R = \frac{\partial \lambda_i}{\partial R}$$

Demonstration for the Roessler system (II)

Extending the matrices and parameters:

$$s_{ij}^{k} = -H_0 \frac{\langle v_i | \kappa (b e_j^T) | u_i \rangle}{\langle v_i | D | u_i \rangle}$$

$$s_i^{R} = -\frac{H_0 \mu_i^{-1}}{(1 - R\mu_i^{-1})} \frac{\langle v_i | \kappa (b k^T) | u_i \rangle}{\langle v_i | D | u_i \rangle}$$

$$D = I + H_1 \tau \kappa (b k^T)$$

$$H_0 = \frac{1 - \mu_i^{-1}}{1 - R\mu_i^{-1}},$$

$$H_1 = \frac{\mu_i^{-1}(1 - R)}{(1 - R\mu_i^{-1})^2}$$

$$p = (k, R)$$

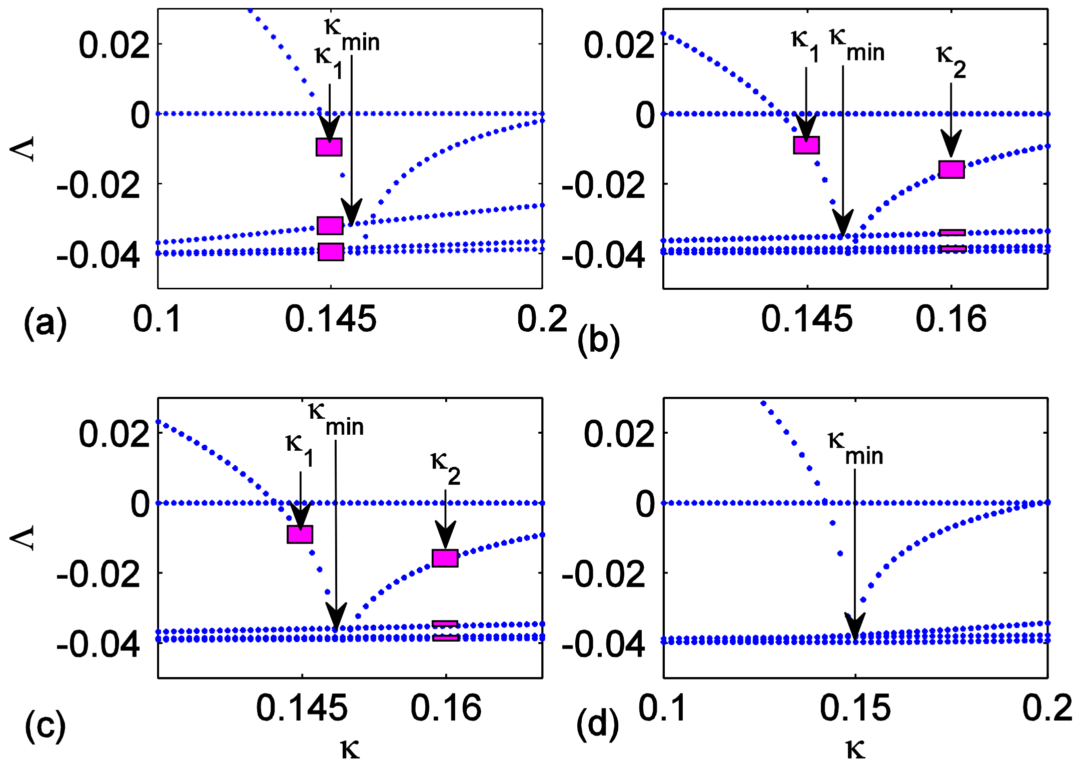
$$S = (S_k, S_R)$$

$$\Delta\Lambda = S\Delta p$$

$$\Delta p = S^{+}\Delta\Lambda$$

Demonstration for the Roessler system (III)

Evolution of the shifted roots:



Conclusions

- We have proposed a pole placement method for periodic states controlled by TDFC /ETDFC algorithms. The method enables to deepen the already known minima of the real parts of FEs., i.e. the optimized set of control parameters can be found.

- The method may be straightforwardly generalized to the other modifications of TDFC, e.g., to the NTDFC scheme.

- The method can be used for other optimization problems, e.g., for the broadening the interval of stability of controlled UPO.

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