Search for optimal signals to suppress synchronization in oscillatory networks

Irmantas Ratas (irmantas.ff.vu@gmail.com), Kestutis Pyragas

Center for Physical Sciences and Technology
A. Gostauto 11, LT-01108 Vilnius
LITHUANIA

8th European Nonlinear Dynamics Conference, Vienna, 2014

Outline

- Motivation
- Problem formulation
- Results
- Conclusion

Motivation

- Pathological synchronization - symptoms of neurological diseases
- Desynchronization methods:

  I) open loop (e.g. coordinates reset, high frequency stimulation)

  II) closed loop (e.g. PID, delayed feedback, act-and-wait)

- Our aim is to investigate the possibility to suppress synchronization in oscillatory networks via an external time-dependent force without the use of the feedback.

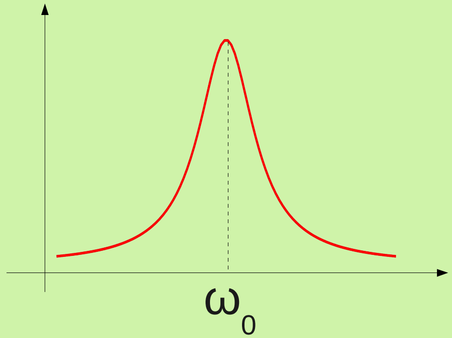
Oscillatory network

Kuramoto model

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j)$$

coupling

ω_j are distributed by Lorentzian



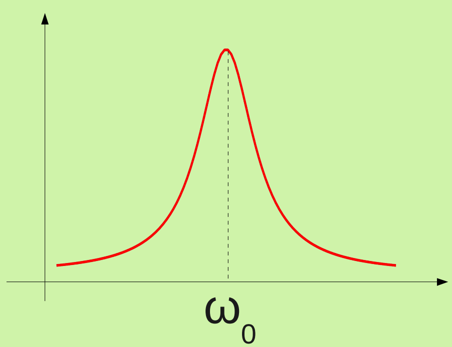
Oscillatory network

Kuramoto model

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N} \sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

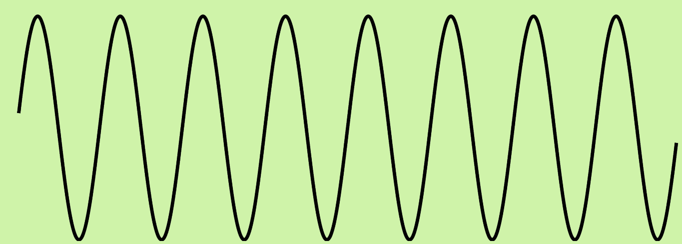
coupling — external force

ω_j are distributed by Lorentzian

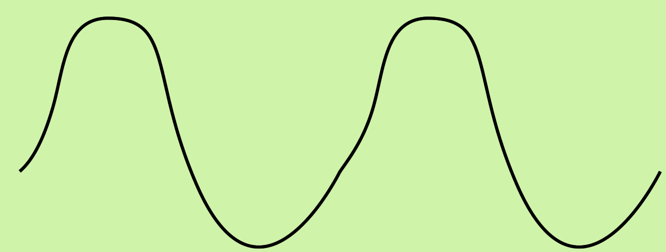


External force is product of two periodic functions:

$\sin(\omega_0 t)$



$A(t) = A(t+T)$



Synchronization estimation

Kuramoto model

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

System synchronization is defined by the ***order parameter***:

$$r = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

Synchronization estimation

Kuramoto model

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

System synchronization is defined by the ***order parameter***:

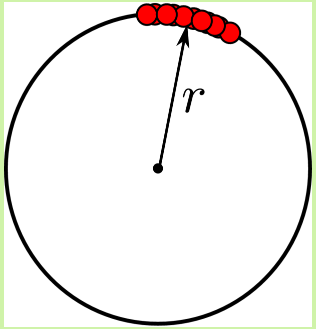
$$r = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

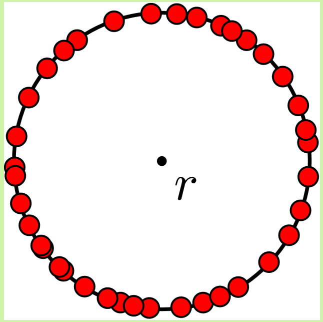
$|r| = 1$

synchronized state

$|r| = 0$

desynchronized state





Synchronization estimation

Kuramoto model

$$\dot{\theta}_j = \omega_j + \frac{K}{N} \sum_{k=1}^{N} \sin(\theta_k - \theta_j) + A(t) \sin(\omega_0 t - \theta_j)$$

System synchronization is defined by the ***order parameter***:

$$r = \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j}$$

$$|r| = 1$$

synchronized state

$$|r| = 0$$

desynchronized state

Object is to minimize |*r*| average through *A(t)* period T

Synchronization estimation

Kuramoto model

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N} \sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

System synchronization is defined by the ***order parameter***:

$$r = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

$$|r| = 1$$

synchronized state

$$|r| = 0$$

desynchronized state

Object is to minimize |*r*| average through *A(t)* period T

What kind of dynamics govern order parameter?

Synchronization estimation

Ott-Antonsen ansatz[1] → order parameter in rotating coordinates satisfy equation:

$$\dot{r} = -\frac{1}{2} r^2 \left(K r^* + A(t) \right) + \frac{1}{2} \left(K r + A(t) \right) - r \Delta$$

[1] Edward Ott and Thomas M. Antonsen, Chaos, 18:037113, 2008

Synchronization estimation

Ott-Antonsen ansatz[1] → order parameter in rotating coordinates satisfy equation:

$$\dot{r} = -\frac{1}{2}r^2\left(Kr^* + A(t)\right) + \frac{1}{2}\left(Kr + A(t)\right) - r\Delta$$

Or in rectangular coordinates $r = x + iy$

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= -\frac{1}{2}\left(\left(x^2 - y^2\right)\left(A + Kx\right) + 2Kxy^2\right) + \frac{1}{2}\left(K - 2\Delta\right)x + \underline{\frac{1}{2}A},\\
\frac{\mathrm{d}y}{\mathrm{d}t} &= \left[-x\left(Kx + A\right) + \frac{K}{2}\left(x^2 - y^2\right) + \frac{K}{2} - \Delta\right]\underline{y}.
\end{aligned}$$

y=0 is stationary independently on *A(t)*.

We need such force that it would stabilize y=0, then *x(t)* would define order parameter.

[1] Edward Ott and Thomas M. Antonsen, Chaos, 18:037113, 2008

Mathematical problem formulation

Minimize functional:

$$J[x] = \int_0^T x^2(t)\mathrm{d}t$$

Mathematical problem formulation

Minimize functional:

$$J[x] = \int_0^T x^2(t)\mathrm{d}t$$

With constrains:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{2}x^2\left(A + Kx\right) + \frac{1}{2}\left(Kx + A\right) - x\Delta,$$

$$0 > \int_0^T \left(\frac{1}{2}K\left(1 - 3x^2\right) - Ax - \Delta\right)\mathrm{d}t, \quad \text{x(t)=x(t+T) stability}$$

$$0 > \int_0^T \left(\frac{1}{2}K\left(1 - x^2\right) - Ax - \Delta\right)\mathrm{d}t, \quad \text{y=0 stability}$$

$$W = \int_0^T A^2(t)\mathrm{d}t = const., \quad \text{finite power}$$

Mathematical problem formulation

Minimize functional:

$$J[x] = \int_0^T x^2(t)\mathrm{d}t$$

With constrains:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{2}x^2\left(A + Kx\right) + \frac{1}{2}\left(Kx + A\right) - x\Delta,$$

$$0 > \int_0^T \left(\frac{1}{2}K\left(1 - 3x^2\right) - Ax - \Delta\right)\mathrm{d}t,$$ x(t)=x(t+T) stability

$$0 > \int_0^T \left(\frac{1}{2}K\left(1 - x^2\right) - Ax - \Delta\right)\mathrm{d}t,$$ y=0 stability

$$W = \int_0^T A^2(t)\mathrm{d}t = const.,$$ finite power

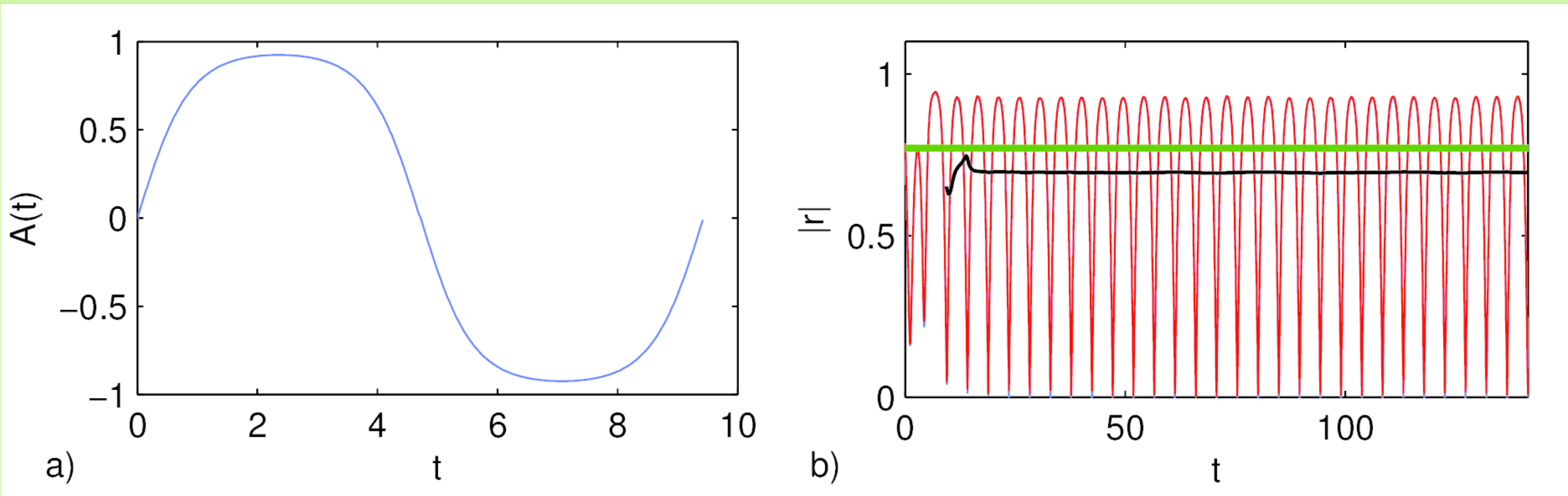
Solution: undefined Lagrange multipliers + numerical integration.

Results

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

Example of external force amplitude

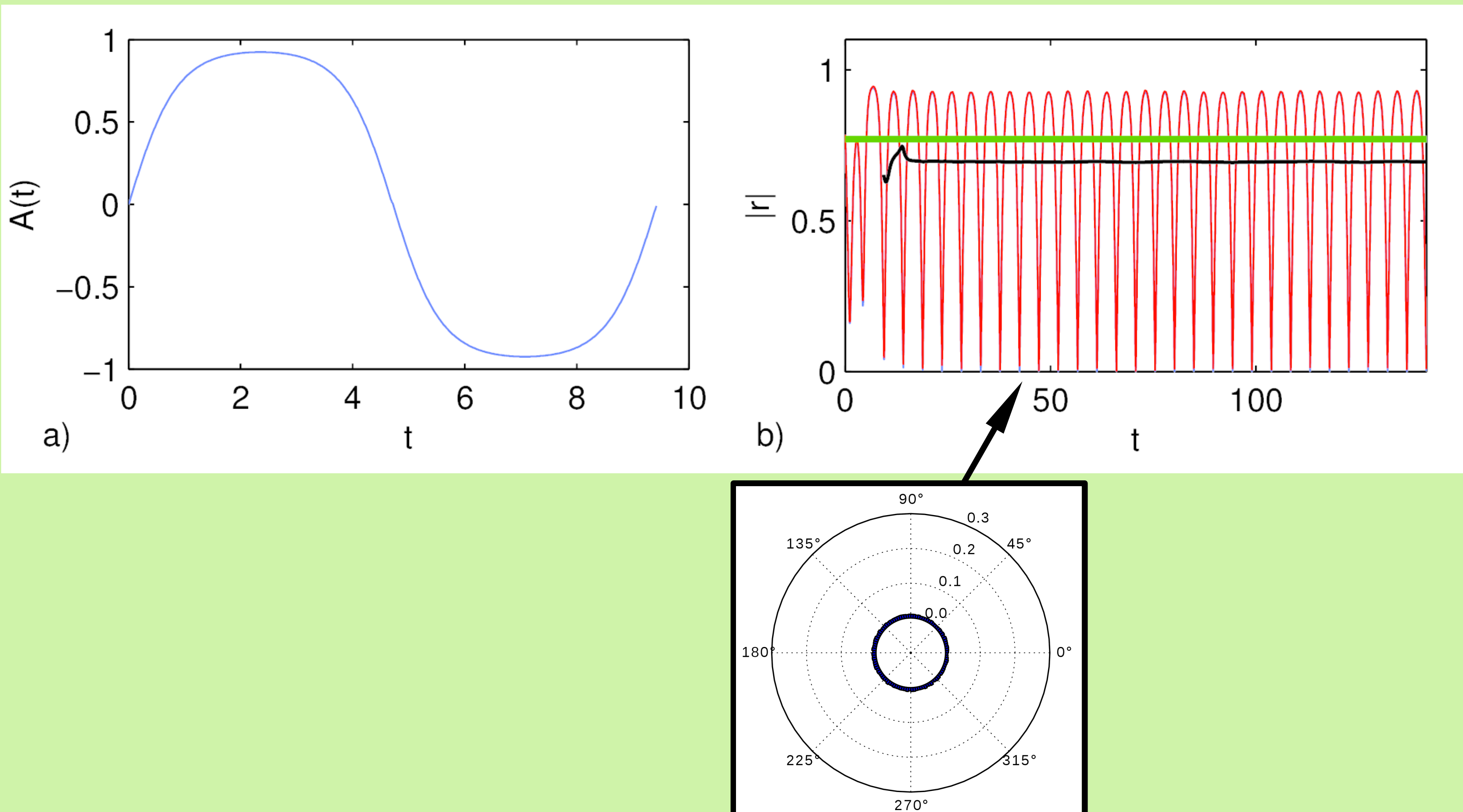
Phase oscillators under control



Results

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

Example of external force amplitude **Phase oscillators under control**

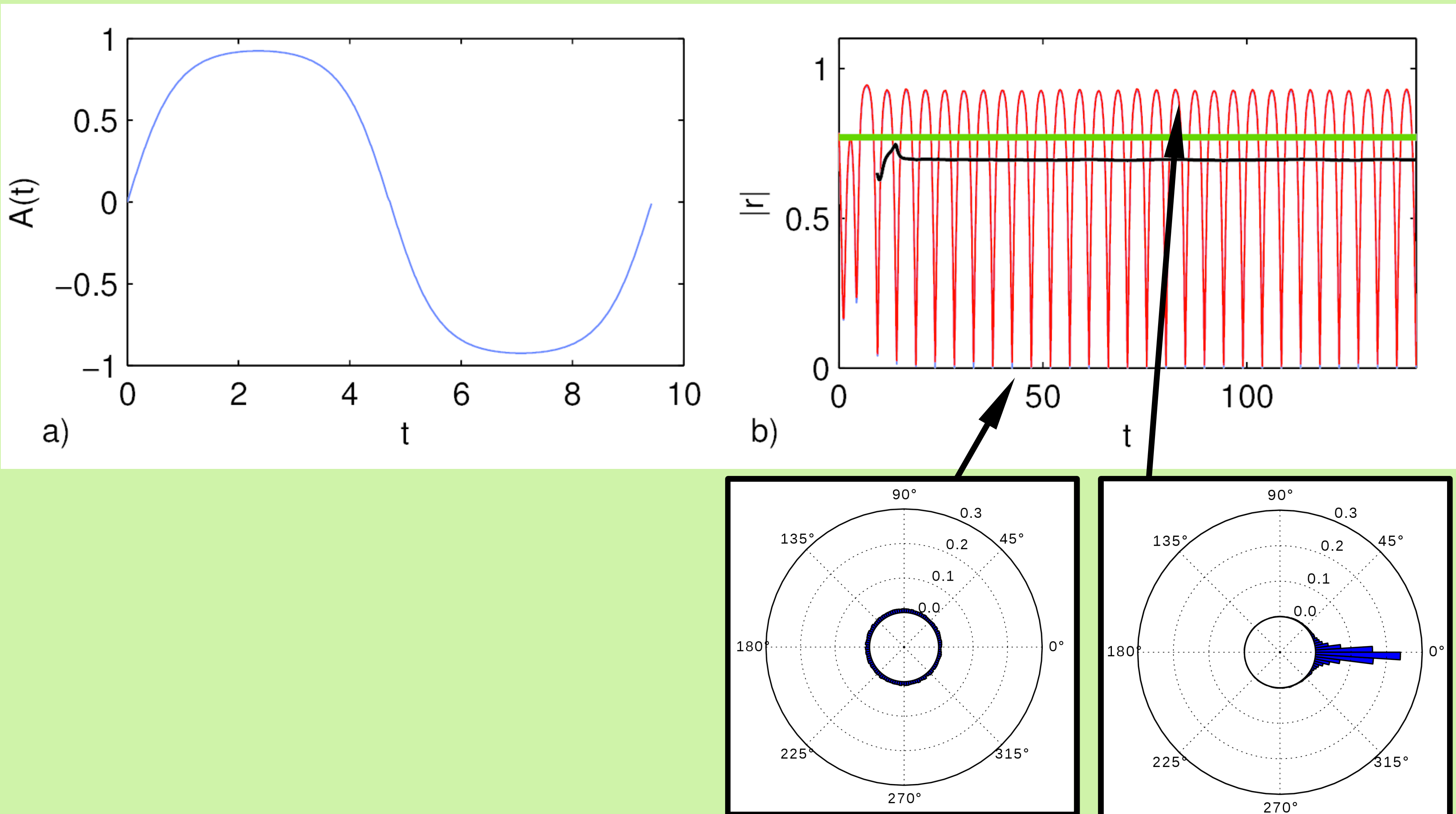


Results

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

Example of external force amplitude

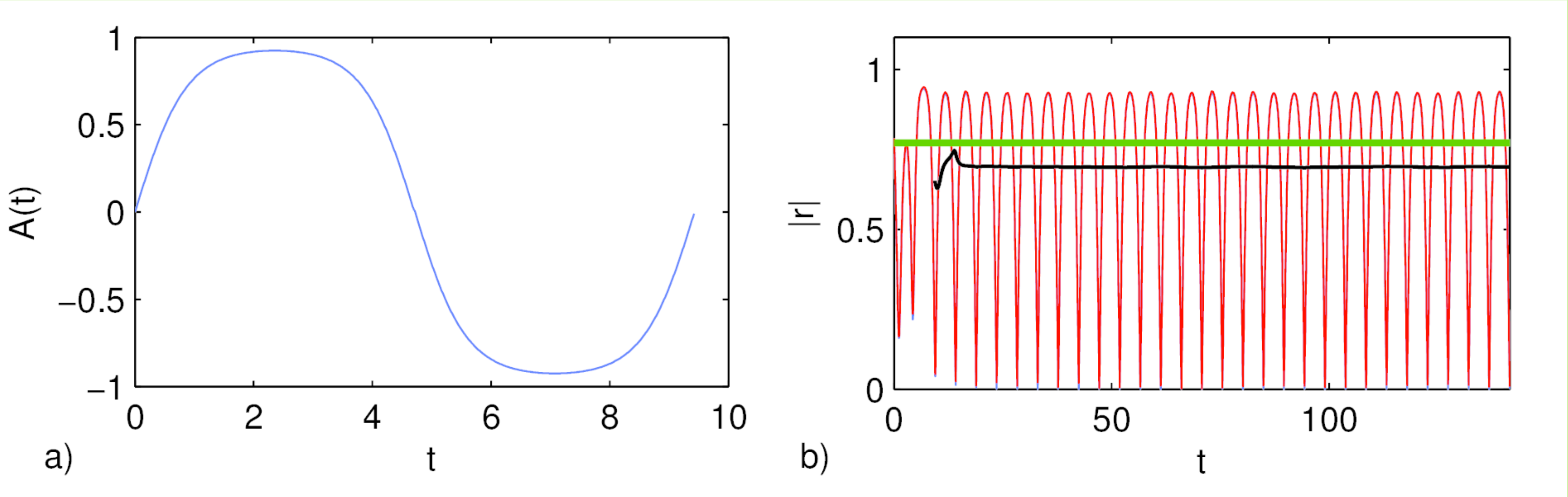
Phase oscillators under control



Results

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

Example of external force amplitude **Phase oscillators under control**

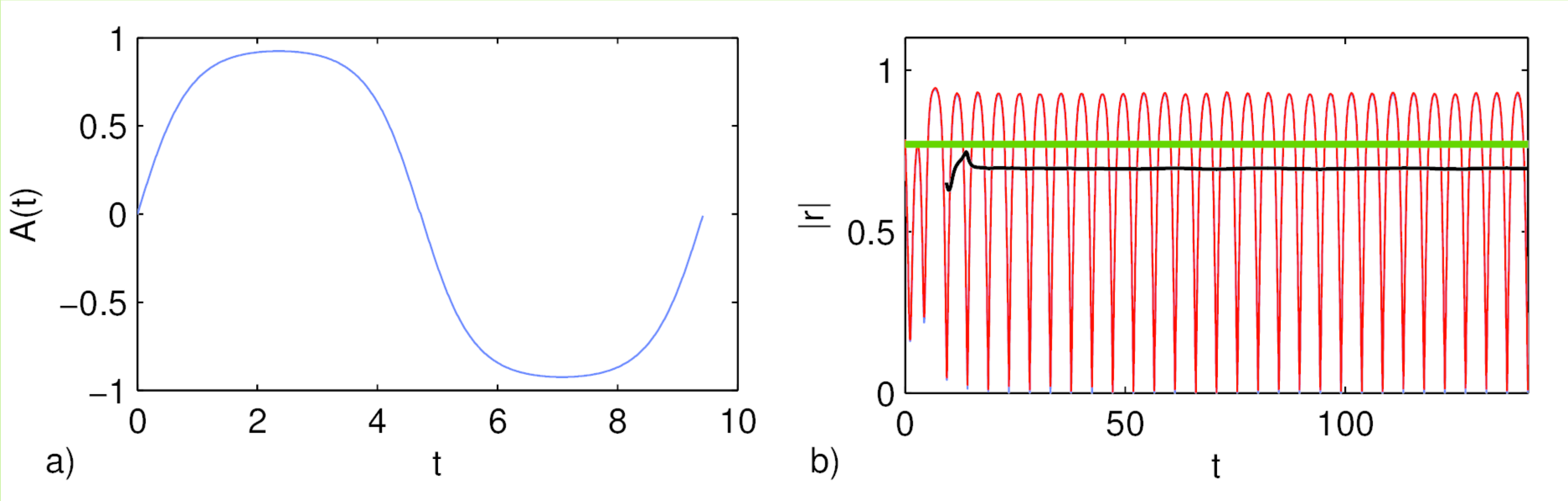


Stimulation is not effective

Results

$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

Example of external force amplitude **Phase oscillators under control**

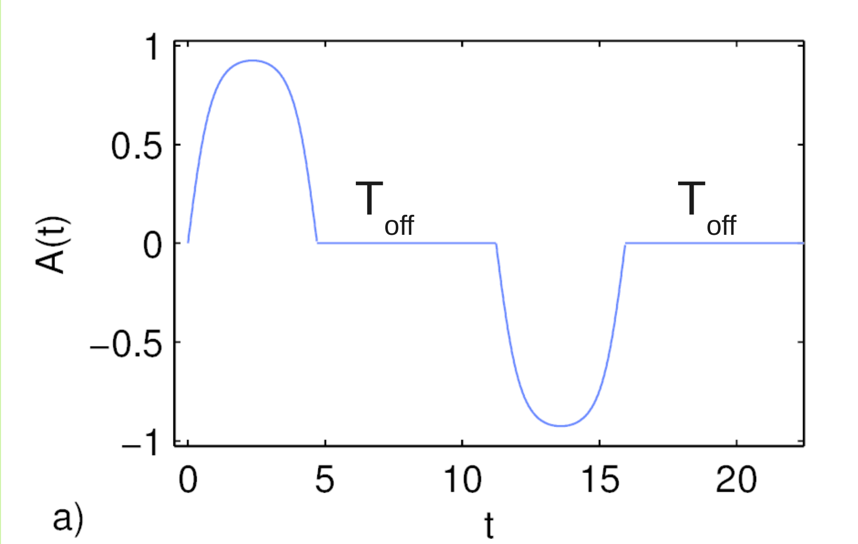


Stimulation is not effective

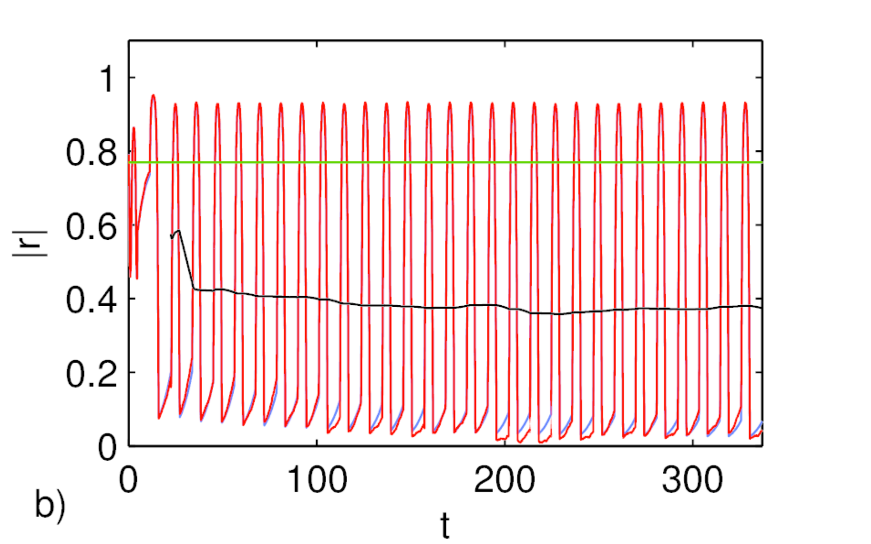
For control free system |*r*|=*0* is <u>unstable stationary point</u>.

Non-smooth amplitude

Example of external force amplitude



Phase oscillators under control



$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j) + A(t)\sin(\omega_0 t - \theta_j)$$

Conclusions

- Desynchronization with smoothly modulated external force which frequency is equal to system's frequency is not effective.
- Efficiency may be increased by non-smooth amplitude modulation.
- Results may be relevant to systems with bistable order parameter (where at the same time exist stable synchronized and incoherent states).

Conclusions

- Desynchronization with smoothly modulated external force which frequency is equal to system's frequency is not effective.
- Efficiency may be increased by non-smooth amplitude modulation.
- Results may be relevant to systems with bistable order parameter (where at the same time exist stable synchronized and incoherent states).

Thank you for attention!

Acknowledgments
This research was funded by the European Social Fund under the Global Grant measure (grant No. VP1-3.1-SMM-07-K-01-025)

The end