Control of synchronization bistability in oscillatory networks

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Outline

- Motivation
- Synchronization estimation
- Synchronization bistability
- Algorithms
- Results
- Conclusion
Motivation

- Synchronization – widely observed phenomena
- Pathological synchronization – symptoms of neurological diseases
- Synchronized state – may be not uniquely stable
- Desynchronization methods:
  - I) open loop (e.g. coordinates reset, high frequency stimulation)
  - II) closed loop (e.g. PID, delayed feedback, act-and-wait)
Synchronization estimation

If oscillator have well defined phase, then synchronization is estimated by the order parameter:

$$r_1 = \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j}$$
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- \( |r_1| = 1 \) synchronized state
- \( |r_1| = 0 \) desynchronized state
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\[ r_1 = \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j} \]

\[ r_n = \frac{1}{N} \sum_{j=1}^{N} e^{in\theta_j} \]

- \(|r_1| = 1\) for synchronized state
- \(|r_1| = 0\) for desynchronized state
Synchronization estimation

In real life to define separate oscillators phase in coupled network is impossible.

Neuron network models have shape

\[
\begin{align*}
\frac{dx_i}{dt} &= F(x_i, y_i) + W(x_1, \ldots, x_N), \\
\frac{dy_i}{dt} &= G(x_i, y_i).
\end{align*}
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Synchronization may be estimated by the **mean potential field variation**:

\[
S = \text{Var} \left[ \frac{1}{N} \sum_{i=1}^{N} x_i \right]
\]
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\]

Synchronization may be estimated by the mean potential field variation:

\[ S \approx 0 \quad \text{desynchronized state} \quad \leftrightarrow \quad S = \text{Var} \left[ \frac{1}{N} \sum_{i=1}^{N} x_i \right] \quad \rightarrow \quad S \approx \text{Var} [x_i] \quad \text{synchronized state} \]
Synchronization bistability

Incoherent and partially synchronized stable states coexist at a particular range of parameters.
Synchronization bistability

Incoherent and partially synchronized stable states coexist at a particular range of parameters.

Example:


\[
\dot{\theta}_j = \omega_j + \frac{K_1}{N} \sum_{k=1}^{N_1} \sin(\theta_k - \theta_j) - \frac{K_2}{N} \sum_{k=N_1+1}^{N} \sin(\theta_k - \theta_j)
\]

\(\text{conformists}\)

\(\text{contrarians}\)
Synchronization bistability

Incoherent and partially synchronized stable states coexist at a particular range of parameters.

Example:

"Kuramoto Model of Coupled Oscillators with Positive and Negative Coupling Parameters: An Example of Conformist and Contrarian Oscillators" by H. Hong and S. H. Strogatz (PRL, 2011)

\[
\dot{\theta}_j = \omega_j + \frac{K_1}{N} \sum_{k=1}^{N_1} \sin(\theta_k - \theta_j) - \frac{K_2}{N} \sum_{k=N_1+1}^{N} \sin(\theta_k - \theta_j)
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Synchronization bistability

Other examples for phase oscillators:

- Bimodal frequencies distributions\(^1\)
- Kuramoto-Sakaguchi model\(^2\)
- Scale free network\(^3\)

\[ \dot{\theta}_j = \omega_j + \frac{K}{N} \sum_{k=1}^{N} \sin(\theta_k - \theta_j) \]

Frequencies distributed by the sum of two Lorentz distr.

Synchronization bistability

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\[
\dot{\theta}_j = \omega_j + \frac{K}{N} \sum_{k=1}^{N} \sin(\theta_k - \theta_j + \alpha)
\]

Some specific unimodal frequencies distributions

Synchronization bistability

Other examples for phase oscillators:

- Bimodal frequencies distributions\(^1\)
- Kuramoto-Sakaguchi model\(^2\)
- Scale free network\(^3\)

\[^1\] E. Martens et al., "Exact results for the Kuramoto model with a bimodal frequency distribution", Phys. Rev. E, (2009)
Synchronization bistability

FitzHugh-Nagumo synaptically coupled neurons [“conformists-contrarian” analogue]:

\[
\begin{align*}
\dot{v}_j &= v_j \left(1 - \frac{v_j^2}{3}\right) - u_j + I + I_j^{(\text{syn})}, \\
\dot{u}_j &= \varepsilon_j \left(b_0 + b_1 v_j - u_j\right), \\
I_j^{(\text{syn})} &= (v_j^{(0)} - v_j) \frac{K_j}{N} \sum_k \frac{1}{1 + \exp\left(-\frac{(v_k - v_{T_j})}{\Delta}\right)}
\end{align*}
\]

\(k\)th neuron acts on \(j\)th only when exceeds some threshold.

<table>
<thead>
<tr>
<th></th>
<th>Excitatory</th>
<th>Inhibitory</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_j)</td>
<td>0.425</td>
<td>1.275</td>
</tr>
<tr>
<td>(v_j^{(0)})</td>
<td>2.5</td>
<td>-2.5</td>
</tr>
</tbody>
</table>
Algorithms

- Systems synchronization and phase reversion by $\pi$ with external periodic force

$$F(t) = a \cos(\Omega t + \phi(t))$$

The moment of external force disconnection should be determined empirically.

- Stimulate system with high frequency periodic signal with decaying amplitude

$$F(t) = a \exp(-t/\tau) \cos(\tilde{\Omega} t)$$
“Conformists-contrarians” model [phase change]:

$$\dot{\theta}_j = \omega_j + \frac{1}{N} \sum_{k=1}^{N} K_k \sin(\theta_k - \theta_j) + a \cos(\Omega t - \theta_j)$$

System:
- Coupling: $K_k \in \{-3, 1\}$
- Number of oscillators: $N = 10000$
- Central frequency: $\omega_0 = 0$

Force parameters:
- Amplitude: $a = 0.2$
- Frequency: $\Omega = 0$
Results

“Conformists-contrarians” model:

\[
\dot{\theta}_j = \omega_j + \frac{1}{N} \sum_{k=1}^{N} K_k \sin(\theta_k - \theta_j) + a \exp(-t/\tau) \cos(\Omega t - \theta_j)
\]

System:

- Coupling: \( K_k \in \{-3, 1\} \)
- Number of oscillators: \( N = 10000 \)
- Central frequency: \( \omega_0 = 0.3 \)

Force parameters:

- Amplitude: \( a = 4 \)
- Frequency: \( \Omega = 2 \)
- Decay time: \( \tau = 200 \)
Results

FitzHugh-Nagumo “conformists-contrarian” model:

Number of neurons: \[ N = 20000 \]
Mean field frequency: \[ \omega_0 \approx \frac{2\pi}{75} \]
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Number of neurons: \( N = 20000 \)

Mean field frequency: \( \omega_0 \approx \frac{2\pi}{75} \)

Phase change: \( F(t) = a \cos(\Omega t + \phi(t)) \)

External force parameters:

\( a = 0.05, \ \Omega = \frac{2\pi}{75} \)
Results

FitzHugh-Nagumo “conformists-contrarian” model:

Number of neurons: \[ N = 20000 \]
Mean field frequency: \[ \omega_0 \approx \frac{2\pi}{75} \]

Phase change: \[ F(t) = a \cos(\Omega t + \phi(t)) \]

Decaying periodic force: \[ F(t) = a \exp(-t/\tau) \cos(\Omega t) \]

External force parameters:
\[ a = 0.05, \ \Omega = \frac{2\pi}{75} \]

External force parameters:
\[ a = 1.5, \ \Omega = \frac{2\pi}{10}, \ \tau = 500 \]
Conclusion

• Proposed algorithms are able to drive synchronized bistable systems to desynchronized state

Further work and unanswered questions:

• How frequent is investigated systems in the nature?
• How to improve algorithms stability? (dependence on number of oscillators, noise, ...)
• How to estimate control parameters?
• etc...
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Thank you for attention!

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