Control of synchronization bistability in oscillatory networks

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Outline

- Motivation
- Synchronization estimation
- Synchronization bistability
- Algorithms
- Results
- Conclusion

Motivation

- Synchronization – widely observed phenomena
- Pathological synchronization – symptoms of neurological dis-eases
- Synchronized state – may be not uniquely stable
- Desynchronization methods:

  I) open loop (e.g. coordinates reset, high frequency stimulation)

  II) closed loop (e.g. PID, delayed feedback, act-and-wait)

Synchronization estimation

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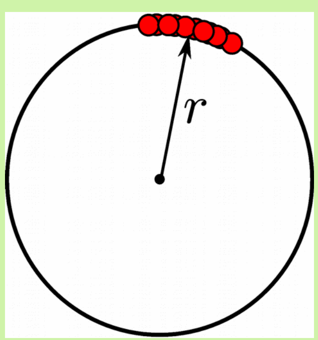
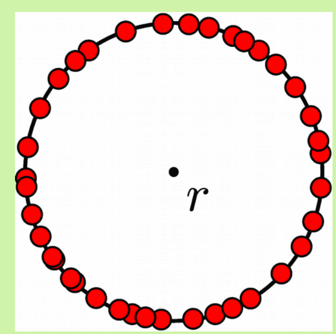
$|r_1| = 1$

synchronized state



$|r_1| = 0$

desynchronized state



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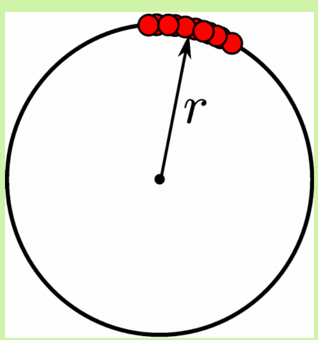

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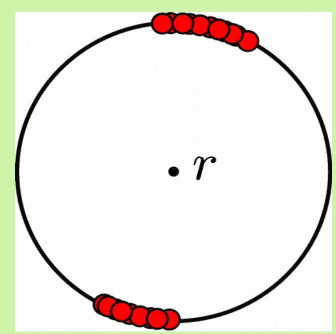

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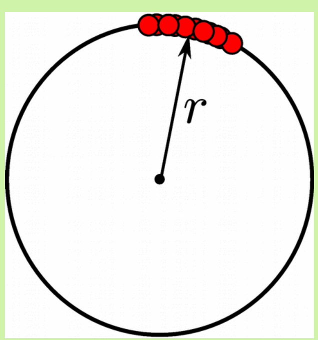
$$r_n = \frac{1}{N}\sum_{j=1}^{N} e^{in\theta_j}$$

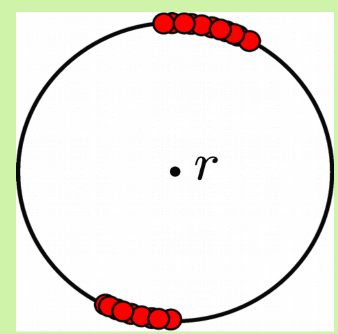
$$|r_1| = 1$$

synchronized state

$$|r_1| = 0$$

desynchronized state





Synchronization estimation

In real life to define separate oscillators phase in coupled network is impossible.

Neuron network models have shape

Oscill. Eqs. Coupling

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = F(x_i, \mathbf{y_i}) + W(x_1, ..., x_N),$$

$$\frac{\mathrm{d}\mathbf{y}_i}{\mathrm{d}t} = \mathbf{G}(x_i, \mathbf{y_i}).$$

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Synchronization may be estimated by the **mean potential field variation**:

$$S = \mathrm{Var}\left[\frac{1}{N}\sum_{i=1}^{N} x_i\right]$$

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$S \approx 0$
desynchronized state

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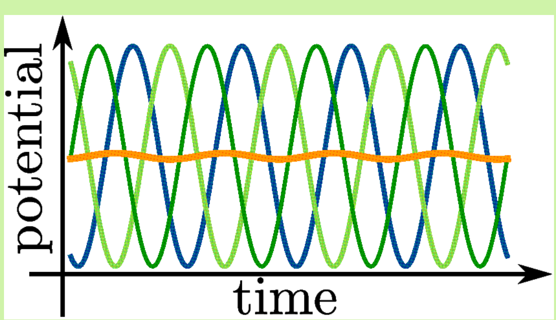
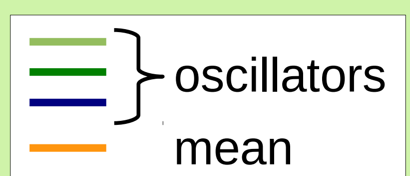
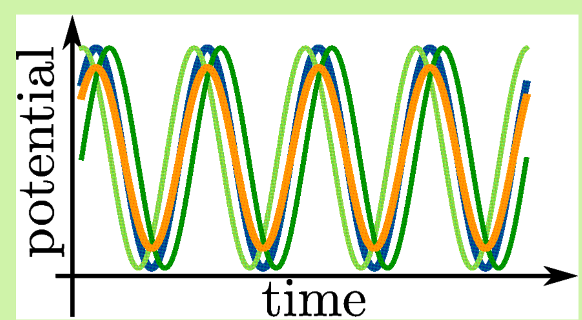
$S \approx \mathrm{Var}\left[x_i\right]$
synchronized state







Synchronization bistability

Incoherent and partially synchronized stable states coexist at a particular range of parameters.

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Example:

"Kuramoto Model of Coupled Oscillators with Positive and Negative Coupling Parameters: An Example of Conformist and Contrarian Oscillators" by H. Hong and S. H. Strogatz (PRL, 2011)

conformists

$$\dot{\theta}_j = \omega_j + \frac{K_1}{N}\sum_{k=1}^{N_1}\sin(\theta_k - \theta_j) - \frac{K_2}{N}\sum_{k=N_1+1}^{N}\sin(\theta_k - \theta_j)$$

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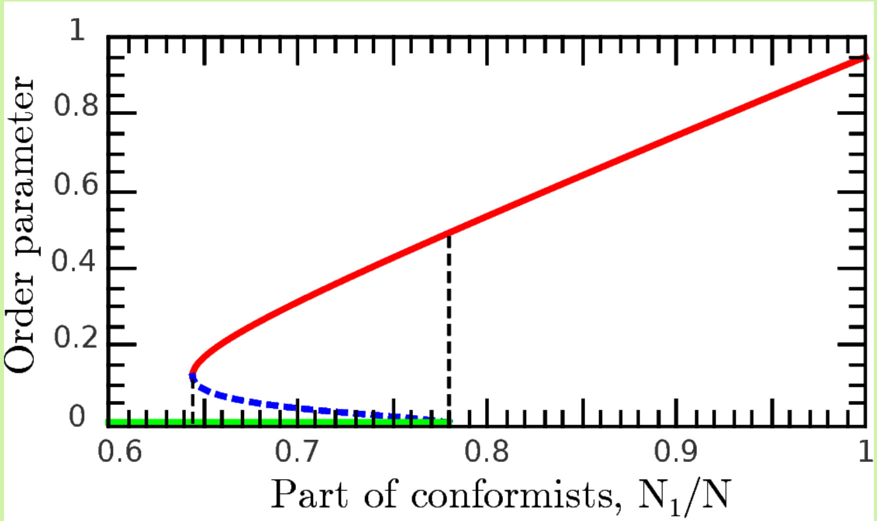


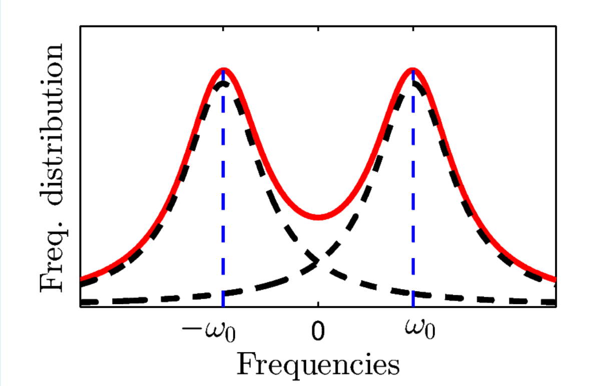
Synchronization bistability

Other examples for phase oscillators:

- Bimodal frequencies distributions[1]
- Kuramoto-Sakaguchi model[2]
- Scale free network[3]

$$\dot{\theta}_j = \omega_j + \frac{K}{N} \sum_{k=1}^{N} \sin(\theta_k - \theta_j)$$

Frequencies distributed by the sum of two Lorentz distr.



[1] E. Martens et al. ,"Exact results for the Kuramoto model with a bimodal frequency distribution", Phys. Rev. E, (2009)

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$$\dot{\theta}_j = \omega_j + \frac{K}{N}\sum_{k=1}^{N}\sin(\theta_k - \theta_j + \alpha)$$

Some specific unimodal frequencies distributions

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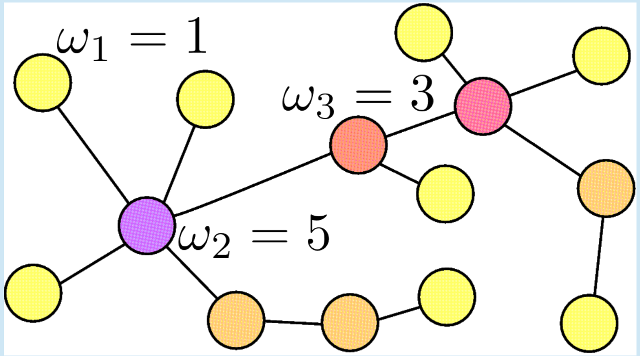
Probability to be attached to k oscillators $\sim k^{-\gamma}$



$$\dot{\theta}_j = \omega_j + K \sum_{k=1}^{N} A_{ij} \sin(\theta_k - \theta_j)$$

$$\omega_j \sim \#\ \text{attached oscillators}$$

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Synchronization bistability

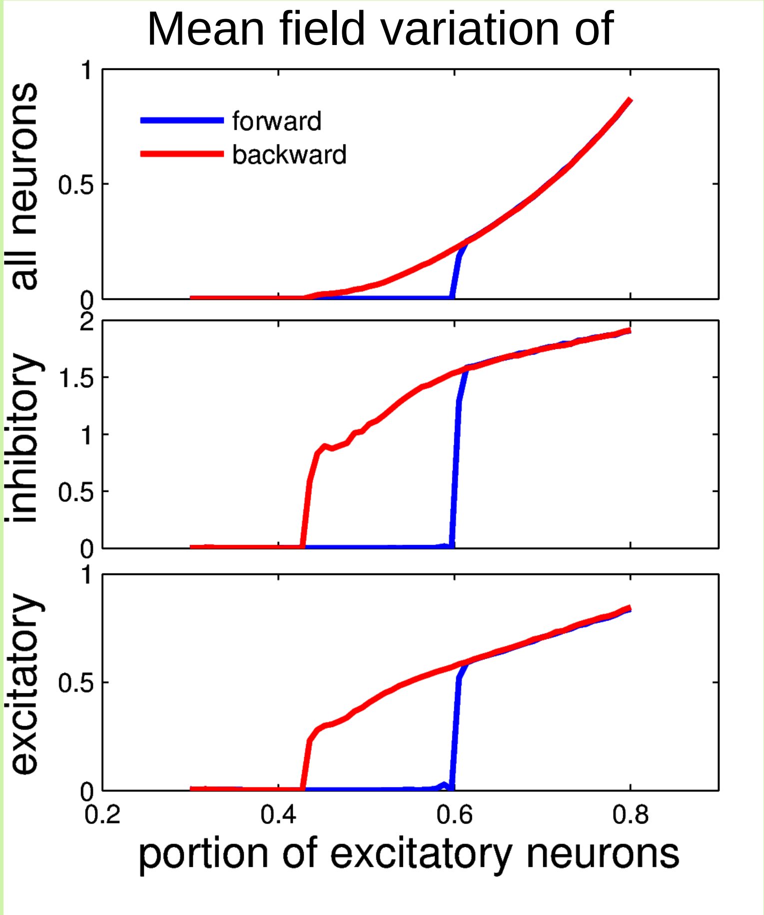
FitzHugh-Nagumo synaptically coupled neurons [“conformists-contrarian” analogue]:

$$\dot{v}_j = v_j(1 - v_j^2/3) - u_j + I + I_j^{(syn)},$$
$$\dot{u}_j = \varepsilon_j(b_0 + b_1 v_j - u_j),$$

$$I_j^{(syn)} = (v_j^{(0)} - v_j)\frac{K_j}{N}\sum_k \frac{1}{1 + \exp(-(v_k - v_T)/\Delta)}$$

k'th neuron acts on *j*'th only when exceeds some threshold.

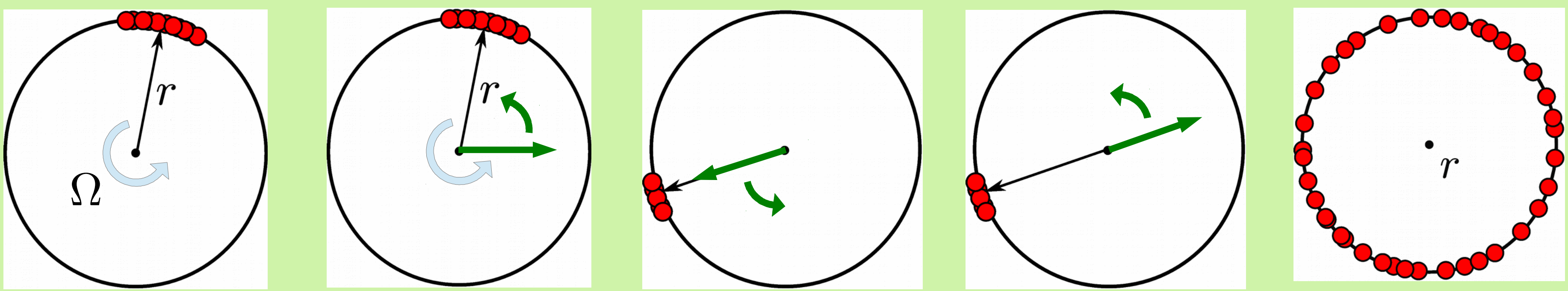
	Excitatory	Inhibitory
K_j	0.425	1.275
$v_j^{(0)}$	2.5	-2.5



Algorithms

- Systems synchronization and phase reversion by π with external periodic force

$$F(t) = a\cos(\Omega t + \phi(t))$$



The moment of external force disconnection should be determined empirically.

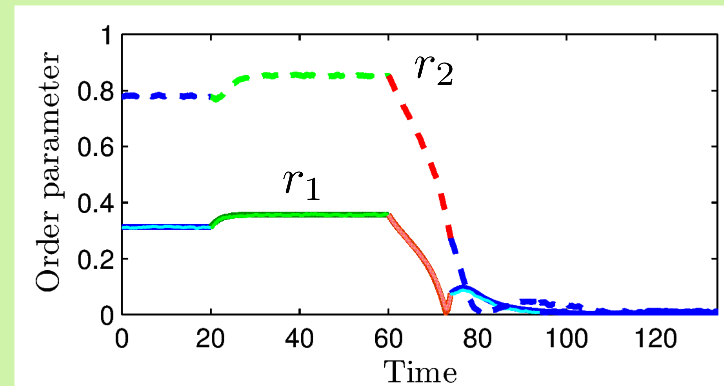
- Stimulate system with high frequency periodic signal with decaying amplitude

$$F(t) = a\exp(-t/\tau)\cos(\bar{\Omega} t)$$

Results

“Conformists-contrarians” model [phase change]:

$$\dot{\theta}_j = \omega_j + \frac{1}{N}\sum_{k=1}^{N} K_k \sin(\theta_k - \theta_j) + a\cos(\Omega t - \theta_j)$$

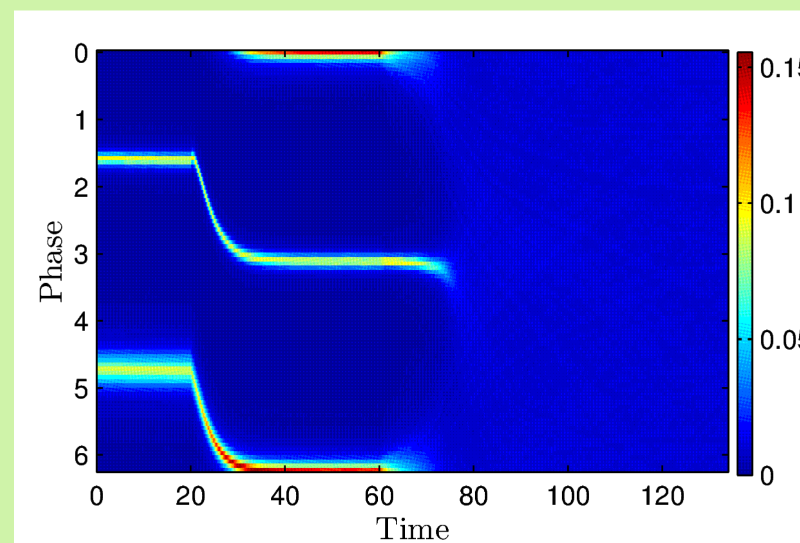


System:

- Coupling $K_k \in \{-3, 1\}$
- Number of oscillators $N = 10000$
- Central frequency $\omega_0 = 0$

Force parameters:

- Amplitude $a = 0.2$
- Frequency $\Omega = 0$



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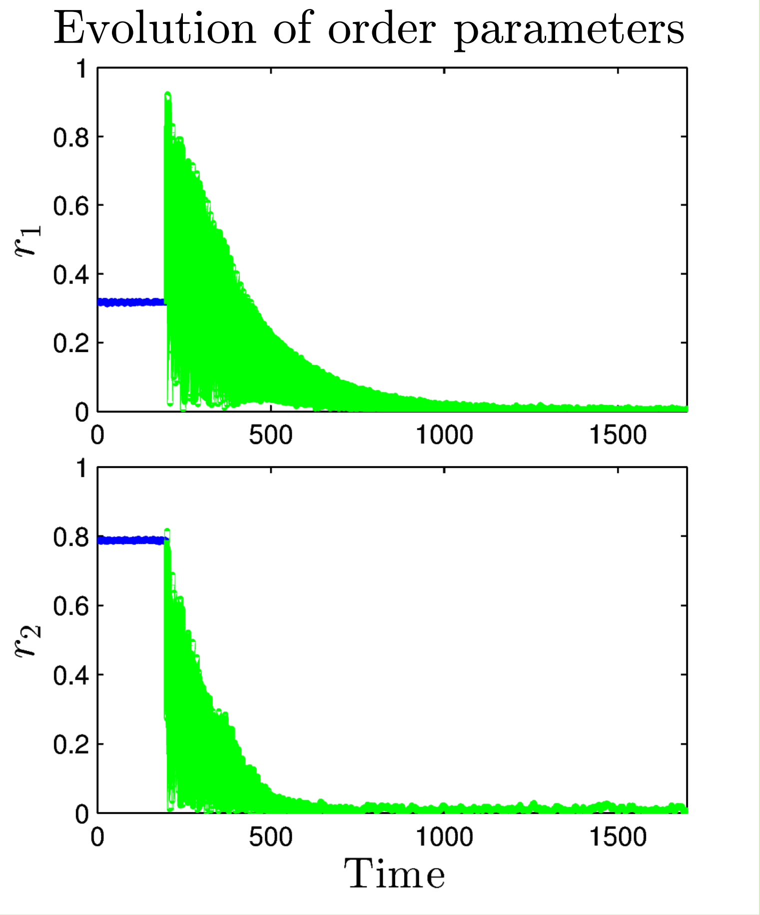

System:

Coupling	$K_k \in \{-3, 1\}$
Number of oscillators	$N = 10000$
Central frequency	$\omega_0 = 0.3$

Force parameters:

Amplitude	$a = 4$
Frequency	$\Omega = 2$
Decay time	$\tau = 200$

Results

FitzHugh-Nagumo “conformists-contrarian” model:

Number of neurons: $N = 20000$

Mean field frequency: $\omega_0 \approx 2\pi/75$

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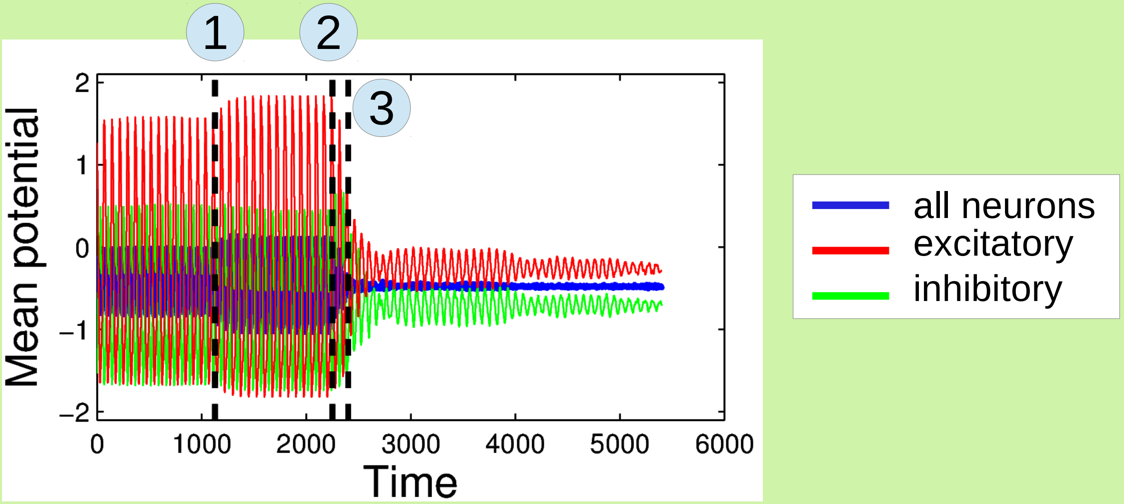
Phase change: $\mathrm{F(t)} = \mathrm{a}\ \cos(\Omega t + \phi(t))$



External force parameters:

$a = 0.05,\ \Omega = 2\pi/75$

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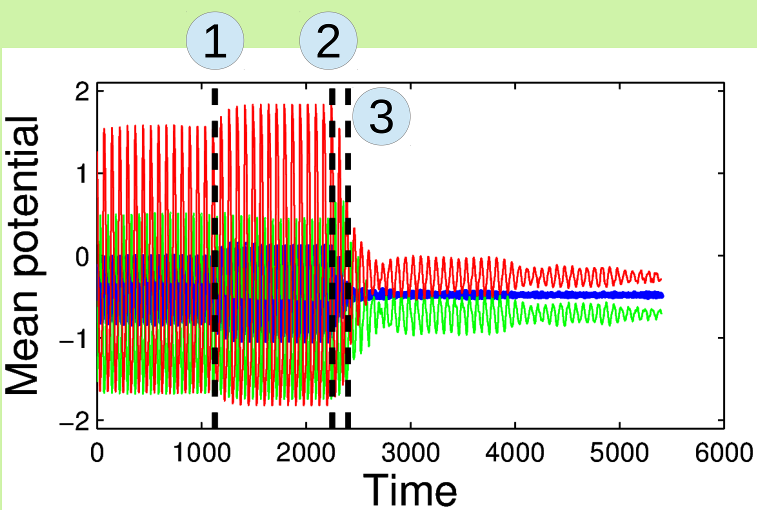

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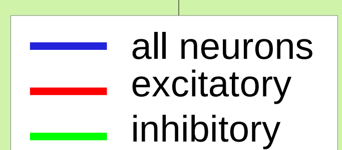
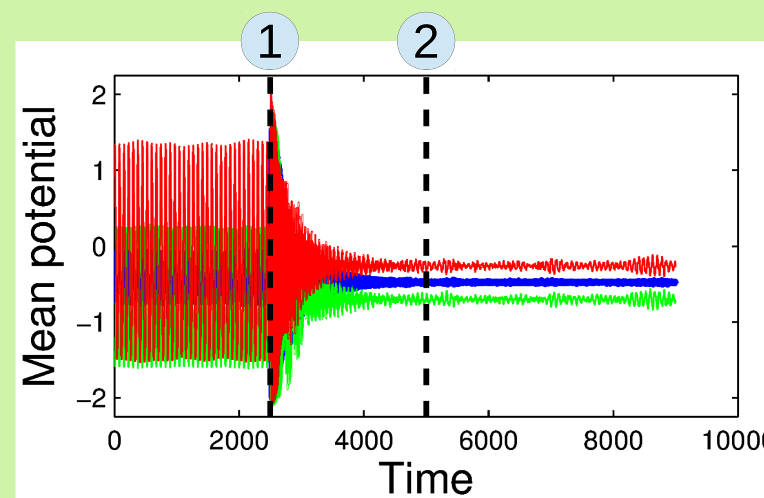


Decaying periodic force: $\mathrm{F(t)}=\mathrm{a}\exp(-t/\tau)\cos(\Omega t)$



External force parameters:

$a = 1.5,\ \Omega = 2\pi/10,\ \tau = 500$

Conclusion

- Proposed algorithms are able to drive synchronized bistable systems to desynchronized state

Further work and unanswered questions:

- How frequent is investigated systems in the nature?
- How to improve algorithms stability? (dependence on number of oscillators, noise, ...)
- How to estimate control parameters?
- etc...

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Thank you for attention!

Acknowledgments
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