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Analytical expression for the period of orbits stabilized by extended delayed feedback control

Viktor Novičenko and Kestutis Pyragas

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Outline

- Introduction to DFC and EDFC techniques
- Delay time is close, but not equal, to period of UPO
- Phase reduction approach
- Period of system perturbed by small value
- Expression for period of orbit stabilized by EDFC
- Example: numerical simulation of Rossler system
- Conclusions

Delay feedback control (DFC) and extended DFC technique (1)

Autonomic system with a unstable periodic orbit (UPO):

$$\dot{X} = F(X)$$

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τ must be equal to the period of UPO

noninvasive control force (DFC case)

K. Pyragas, Continuous control of chaos by self-controlling feedback, Phys. Lett. A 170 (1992) 421–428

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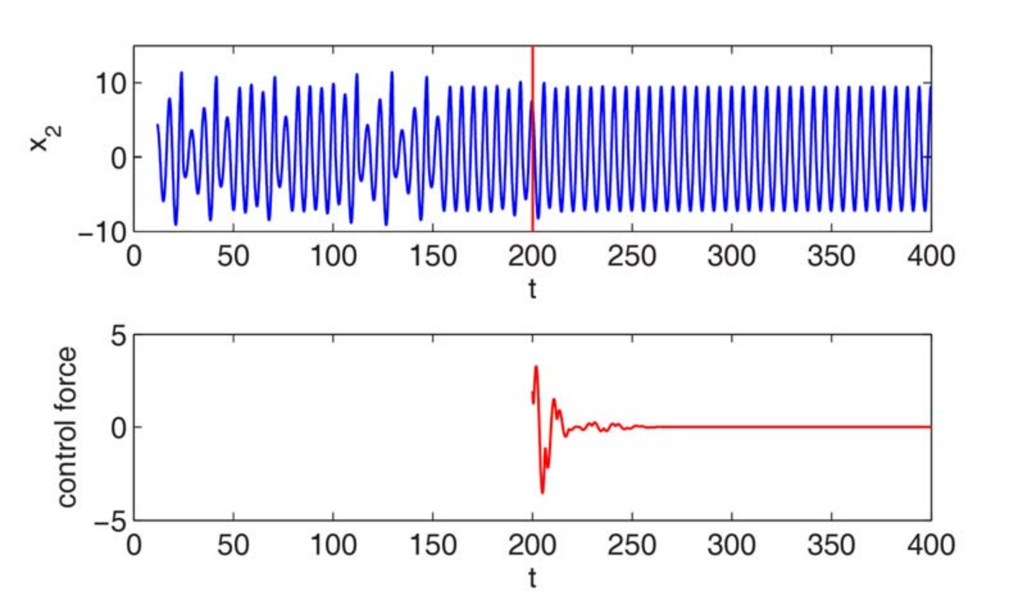
Example of stabilization of period-one UPO in Rossler system:



Delay feedback control (DFC) and extended DFC technique (2)

Autonomic system with a unstable periodic orbit (UPO):

$$\dot{X} = F(X)$$

noninvasive control force (EDFC case)

$$+K\left[\left(x_1(t-\tau)-x_1(t)\right)+R\left(x_1(t-2\tau)-x_1(t-\tau)\right)+R^2\left(x_1(t-3\tau)-x_1(t-2\tau)\right)+\ldots\right]$$

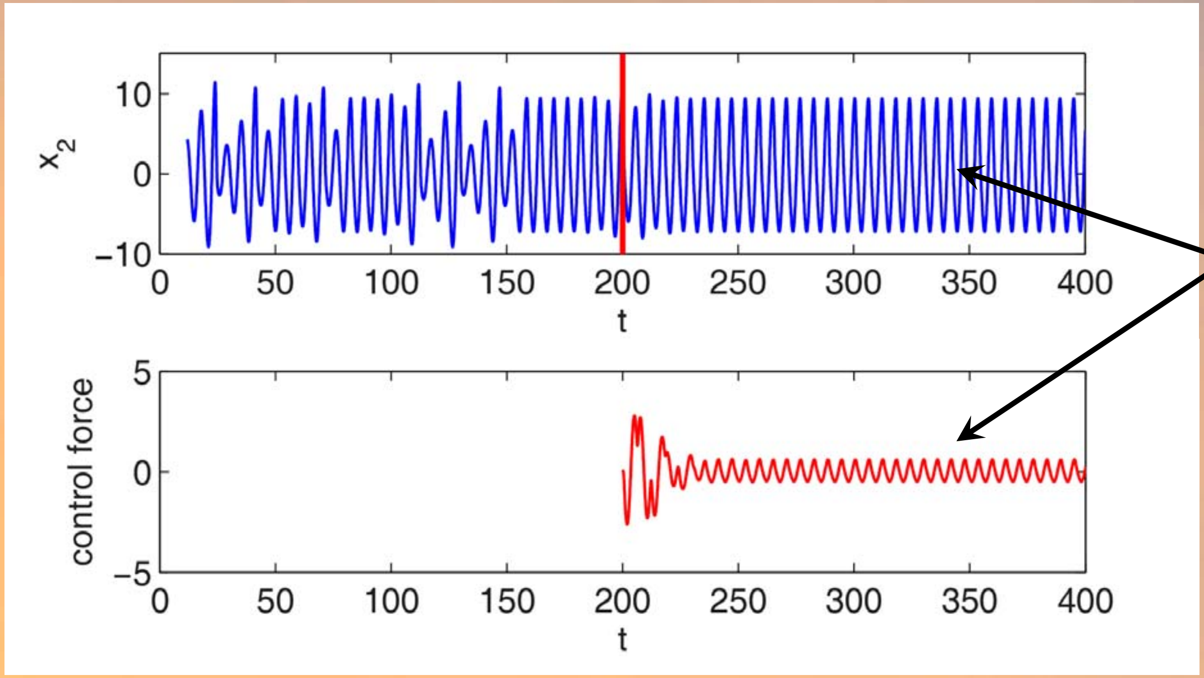
J. E. S. Socolar, D. W. Sukow, and D. J. Gauthier, Stabilizing unstable periodic orbits in fast dynamical systems, Phys. Rew. E 50, 3245 (1994)

If R=0, we get the DFC case.

Some UPO can not be stabilized using the DFC, but can be stabilized using the EDFC.

Delay time is not equal to the period of UPO

Delay time is close, but not equal to the period of UPO:



Oscillations with period Θ, which is something between the delay time and the period of UPO.

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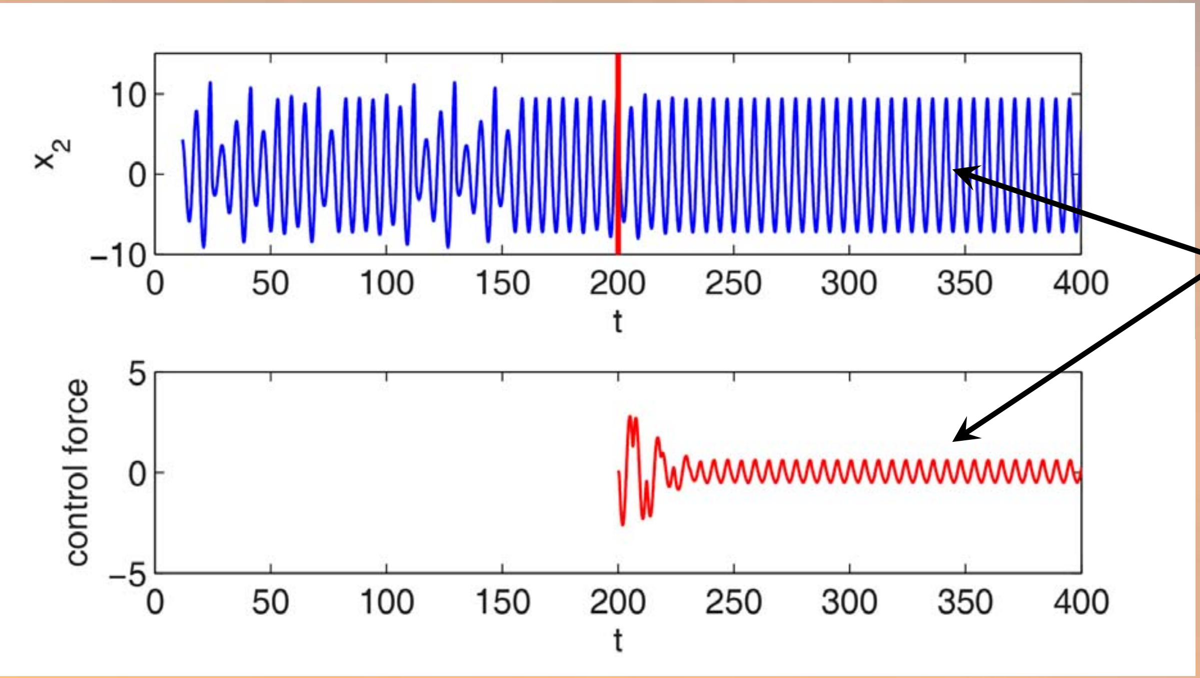


Oscillations with period Θ, which is something between the delay time and the period of UPO.

$$\Theta(K,\tau) = T + \frac{K}{K-\kappa}(\tau - T) + \mathrm{O}\left((\tau - T)^2\right)$$

W. Just, D. Reckwerth, J. Mckel, E. Reibold, and H. Benner, Delayed feedback control of periodic orbits in autonomous systems, Phys. Rew. Lett. 81, 562 (1998)

Introduction to phase reduction (1)

Phase reduction method is an efficient tool to analyze weakly perturbed limit cycle oscillations.

Most investigations in the field of phase reduction are devoted to the systems described by ODEs.

Phase reduction method is extended to delay differential equations (DDE) :

V. Novičenko and K. Pyragas, Phase reduction of weakly perturbed limit cycle oscillations in time-delay systems, Physica D 241, 1090 (2012).

Introduction to phase reduction (2)

A dynamical system with a stable limit cycle.

For each state on the limit cycle and near the limit cycle is assigned a scalar variable (PHASE).
The phase dynamics of the free system satisfies:

$$\dot{\varphi} = 1$$

Let's apply an external perturbation to the system.
The aim of phase reduction method is to find a dynamical equation the phase of perturbed system:

$$\dot{\varphi} = ?$$

Phase reduction of ODE systems

Perturbed system: $\dot{X} = F(X) + \varepsilon\Psi(t)$

Malkin, I.G.: Some Problems in Nonlinear Oscillation Theory.Gostexizdat, Moscow (1956)

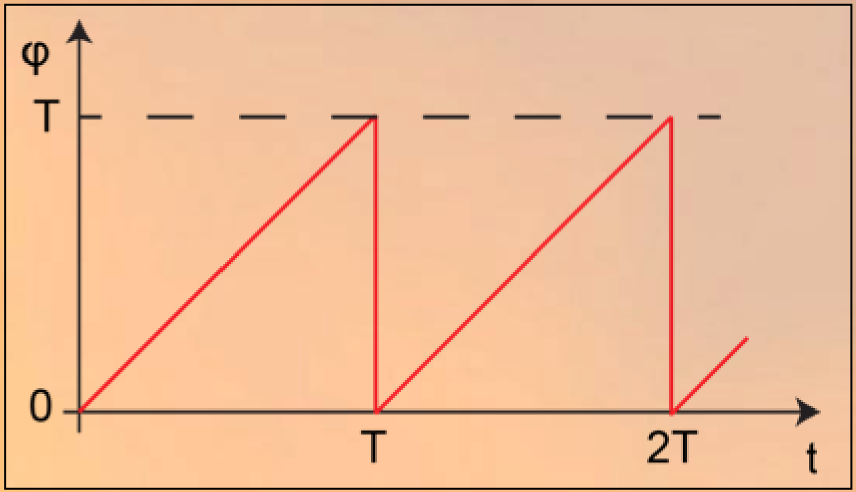
Phase dynamics: $\dot{\varphi} = 1 + \varepsilon Z(\varphi)\Psi(t)$,here $Z(\varphi)$ is periodic vector valued function - the phase response curve (PRC)

PRC is the periodic solution of an adjoint equation: $\dot{Z} = -[DF(X_c)]^T Z$

With initial condition: $Z(0)\dot{X}_c(0) = 1$

Period of system perturbed by a small value

Free system:



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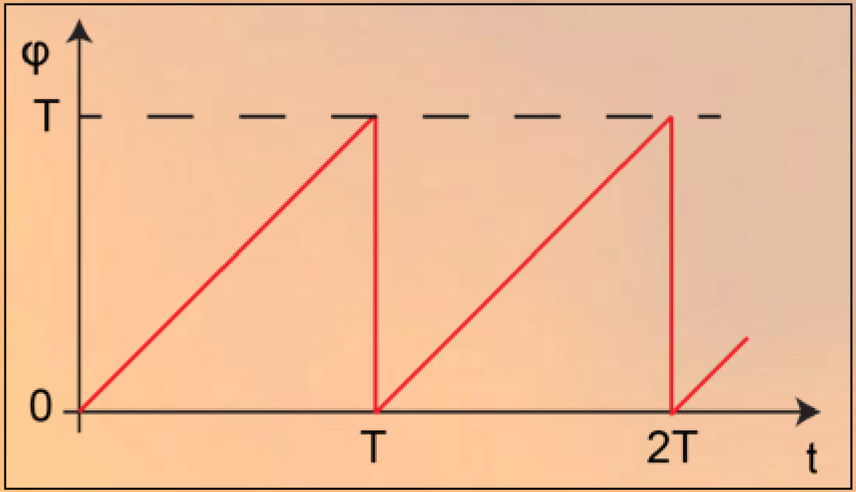
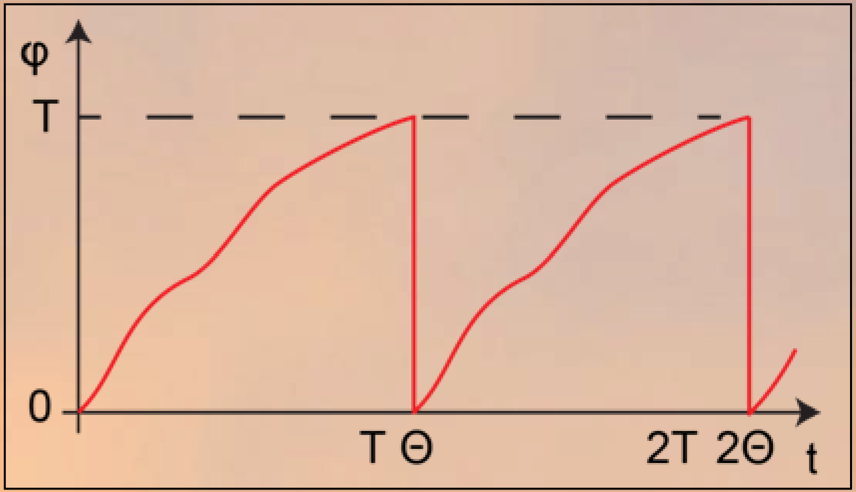


State dependent perturbation:

$$\dot{X} = F(X) + \varepsilon P(X)$$



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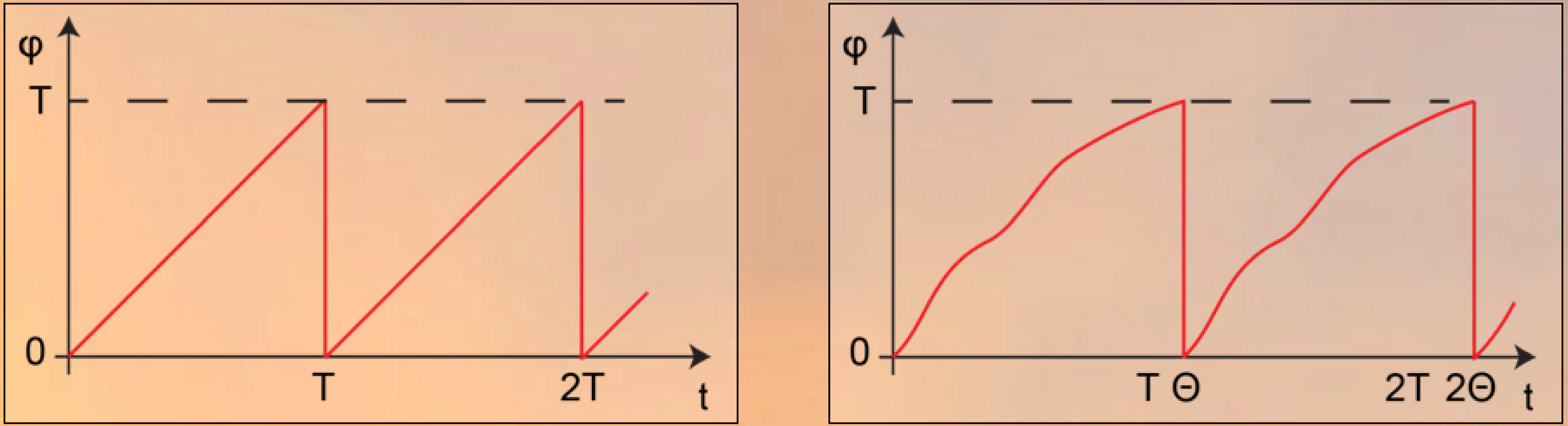

From the phase equation we get a perturbed period:

$$\frac{d\varphi}{dt} = 1 + \varepsilon Z(\varphi) P(X(t)) \Longrightarrow \Theta = T - \varepsilon \int_0^T Z(\varphi) P(X_c(\varphi)) d\varphi$$

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Analytical expression for the period
SPI of CPST

Expression for period of orbit stabilized by EDFC

Delay time ≠ period of the UPO:

$$\dot{X} = F(X) + K\left[x_1(t-\tau) - x_1(t)\right]$$

Expression for period of orbit stabilized by EDFC

Delay time ≠ period of the UPO:

$$\dot{X} = F(X) \quad + K\left[x_1(t - T - (\tau - T)) - x_1(t)\right]$$

Expression for period of orbit stabilized by EDFC

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Non-mismatch component which stabilize the UPO:

$$K\left[x_1\left(t - T\right) - x_1\left(t\right)\right]$$

Mismatch component which can be treated by phase reduction method:

$$-K\dot{x}_1\left(t - T\right)\cdot\left(\tau - T\right)$$

Expression for period of orbit stabilized by EDFC

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$$-K\dot{x}_1\left(t - T\right)\cdot\left(\tau - T\right)$$

$$\Theta(K, R, \tau) = T + \frac{K}{K - \kappa(1 - R)}\left(\tau - T\right) + O\left((\tau - T)^2\right)$$

External parameter can be found from the PRC of the UPO:

$$\kappa = -\left[\int_0^T Z_{UPO}\left(\varphi\right)\dot{x}_1\left(\varphi\right)d\varphi\right]^{-1}$$

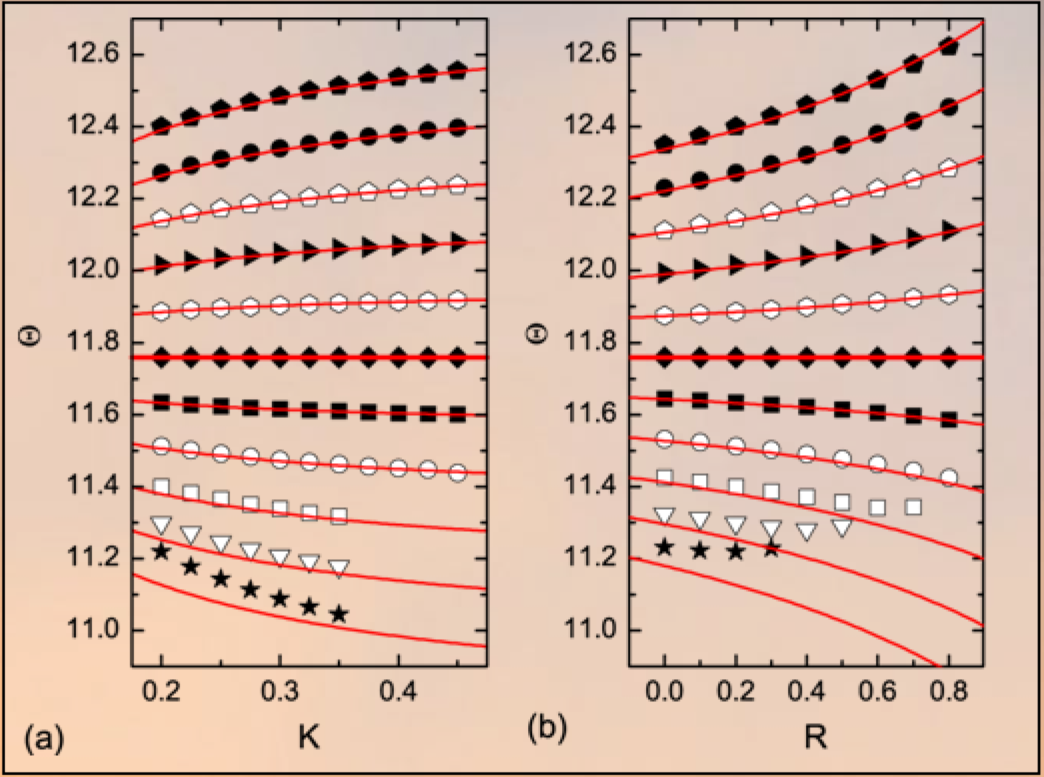
Example: numerical simulation of Rossler system

Rossler system with EDFC:

$$\dot{x}_1 = -x_2 - x_3$$
$$\dot{x}_2 = x_1 + 0.2x_2 + F(t)$$
$$\dot{x}_3 = 0.2 + x_3(x_1 - 5.7)$$

Control force:

$$F(t) = K\left[(1-R)\sum_{j=1}^{+\infty} R^{j-1} x_2(t - j\tau) - x_2(t)\right]$$



(a) R=const=0.2 (b) K=const=0.2

Various mismatch, from bottom to top, τ –T=-1.0; -0.8; -0.6; -0.4; -0.2; 0.0; 0.2; 0.4; 0.6; 0.8; 1.0

Conclusions

- We have considered systems subjected to an extended delayed feedback control force in the case when the delay time differs slightly from the period of unstable periodic orbit of the control-free system
- We have derived an analytical expression which shows in an explicit form how the period of stabilized orbit changes when varying the delay time and the parameters of the control
- Our approach is based on the phase reduction theory adopted to systems with time delay
- The results are useful in experimental implementations, since the unknown period of the UPO can be determined from only few experimental measurements

Acknowledgements

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The end

Phase reduction of time-delay system: results

Phase dynamic: $\dot{\varphi} = 1 + \varepsilon z^T(\varphi)\psi(t)$

The adjoint equation for PRC: $\dot{z} = -A^T(t)z(t) - B^T(t+\tau)z(t+\tau)$

here

$$A(t) = D_1 F\left(x_c(t), x_c(t-\tau)\right)$$
$$B(t) = D_2 F\left(x_c(t), x_c(t-\tau)\right)$$

An unstable difference-differential equation of advanced type (backwards integration)

The initial condition:

$$z^T(0)\dot{x}_c(0) + \int_{-\tau}^{0} z^T(\tau+\vartheta)B(\tau+\vartheta)\dot{x}_c(\tau+\vartheta)d\vartheta = 1$$

The phase reduced equations for time delay systems have been alternatively derive directly from DDE system without appealing to the known theoretical results from ODEs

Phase reduction of chaotic systems subject to a delayed feedback control

System with the stable limit cycle: $\dot{x} = F\left(x(t)\right) + K\left[x(t-\tau) - x(t)\right]$

The adjoint equation for PRC: $\dot{z} = -A^T(t)z(t) - B^T(t+\tau)z(t+\tau)$

$$A(t) = DF\left(x_c(t)\right) - K$$
$$B(t) = K$$

The delay time τ is equal to PRC period, so the adjoint equation can be simplified to:

$$\dot{z} = -\left[DF\left(x_c\right)\right]^T z(t)$$

(Unstable in both directions)

The profile of the PRC is invariant with respect to the variation of K

$$z^{(2)}(\varphi) = \alpha z^{(1)}(\varphi)$$

The coefficient of the proportionality α can be found from the initial condition:

$$\alpha^{-1} = z^{(1)^T}(0)\dot{x}_c(0) + \int_{-\tau}^{0} z^{(1)^T}(\tau + \vartheta)K^{(2)}\dot{x}_c(\vartheta)d\vartheta$$