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Phase anticipating in two forced self - sustained oscillators

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FIZINIŲ IR TECHNOLOGIJOS MOKSLŲ CENTRAS

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EUROPOS SĄJUNGA
Kuriame Lietuvos ateitį

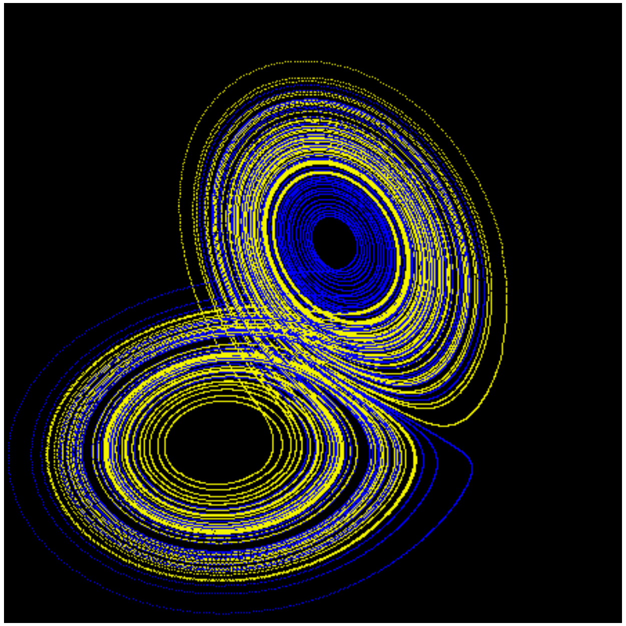
Outline

- Anticipating synchronization
- Phase lead synchronization
- From phase lead to anticipating synchronization
- Conclusions

Chaotical systems

synchronized and controlled (1990)

predicted via anticipating synchronization (2000)



Complete replacement

Anticipating synchronization

$$\dot{\mathbf{r}}_1(t) = -\alpha\,\mathbf{r}_1(t) + \mathbf{f}\left(\mathbf{r}_1(t-\tau)\right)$$ **master**

$$\dot{\mathbf{r}}_2(t) = -\alpha\,\mathbf{r}_2(t) + \mathbf{f}\left(\mathbf{r}_1(t)\right)$$ **slave**

$\alpha > 0$ τ - prediction time

Globally stable solution $\mathbf{r}_2(t) = \mathbf{r}_1(t+\tau)$

Disadvantages

A driver has to possess an internal delay term

Fixed prediction time = driver's delay time

Voss (2000)

Delayed coupling

Anticipating synchronization

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right),$$ **driver without delay**

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + \mathbf{K}[\mathbf{r}_1(t) - \mathbf{r}_2(t-\tau)]$$

Delay is the external parameter

Solution $\mathbf{r}_2(t) = \mathbf{r}_1(t+\tau)$ is stable?

Synchronization regime is stable only for some definite values of delay time and the coupling strength

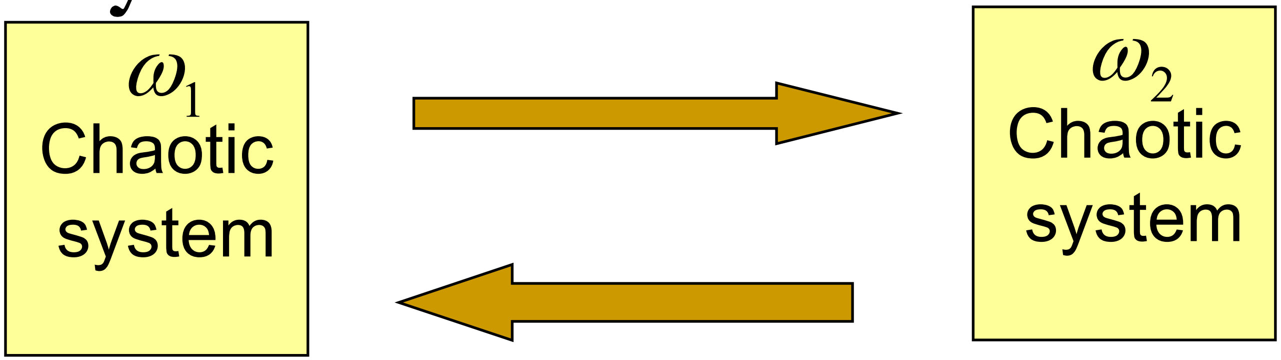
Our former aim was to increase the prediction time

Phil. Trans. R. Soc. A **368**, 305 (2010).

Phys. Rev. E **78,** 046217 (2008).

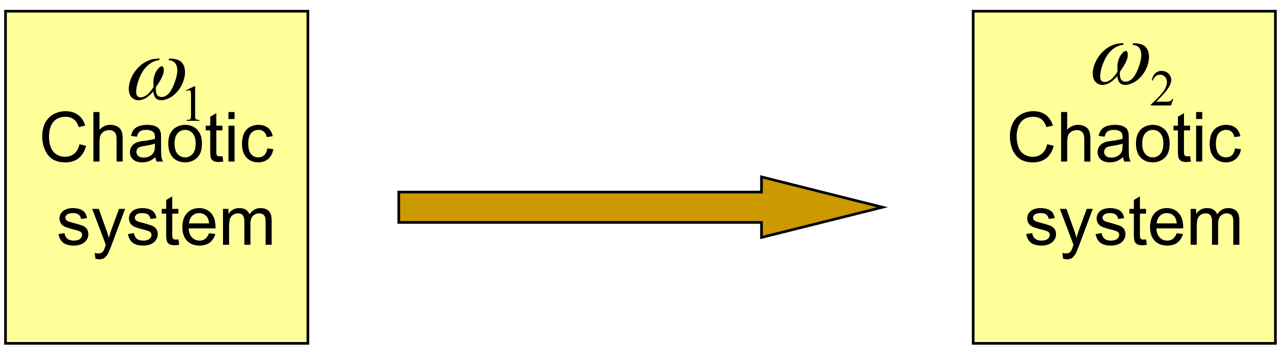
We are seeking a more simple and more universal way for the prediction of chaotic dynamics

Phase synchronization



M. Rosenblum, A. Pikovsky, J. Kurths, PRL **76**, 1804 (1996)
M. Rosenblum, A. Pikovsky, J. Kurths, PRL **78**, 4193 (1997)

How does the phase synchronization manifest itself in unidirectionally coupled systems?



Simple model

$$\dot{\varphi}_1 = \omega_1,$$
$$\dot{\varphi}_2 = \omega_2 + \varepsilon \sin(\varphi_1 - \varphi_2)$$

Adler equation

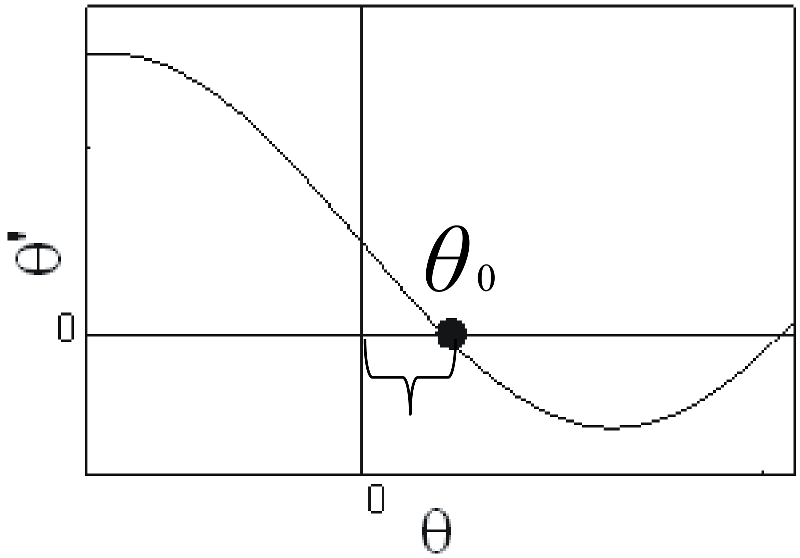
$$\dot{\theta} = \omega_2 - \omega_1 - \varepsilon \sin\theta$$

$$\omega_2 > \omega_1$$

$$\theta = \varphi_2 - \varphi_1$$

Phase lead synchronization (PLS)

$$\dot{\theta}\,\big|_{\theta\,=\,\theta_0} = 0 \qquad \varphi_2 = \varphi_1 + \theta_0$$



$$\theta_0 = \arcsin((\omega_2 - \omega_1)/\varepsilon) > 0 \qquad \omega_2 > \omega_1$$

Two chaotic Rössler systems

Drive system

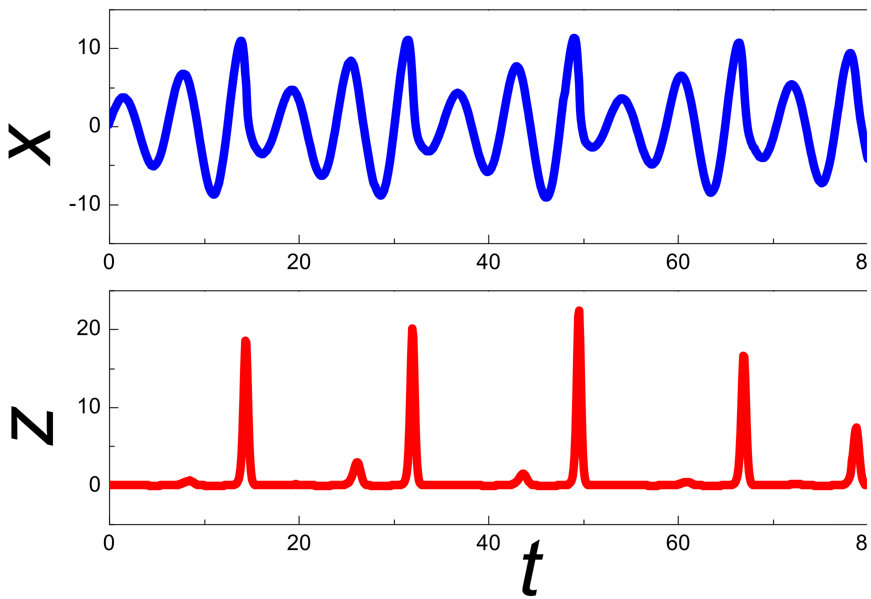
$$\dot{x}_1 = -\omega_1 y_1 - z_1,$$
$$\dot{y}_1 = \omega_1 x_1 + a y_1,$$
$$\dot{z}_1 = b + z_1(x_1 - c).$$

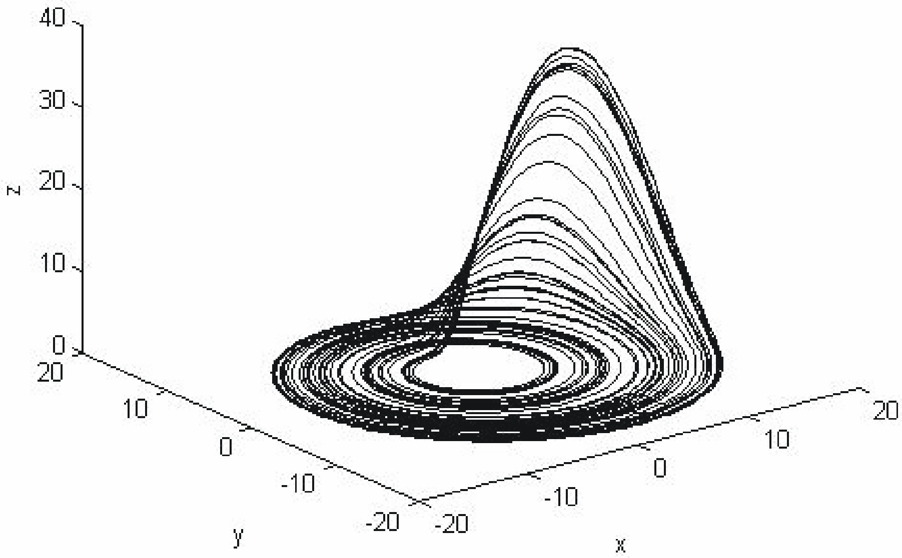
Response system

$$\dot{x}_2 = -\omega_2 y_2 - z_2 + \boxed{\varepsilon(x_1 - x_2)},$$
$$\dot{y}_2 = \omega_2 x_2 + a y_2,$$
$$\dot{z}_2 = b + z_2(x_2 - c).$$

$$a = 0.165,\ b = 0.2,\ c = 10,\ \omega_1 = 0.95,\ \omega_2 = 0.99$$

Phase notion

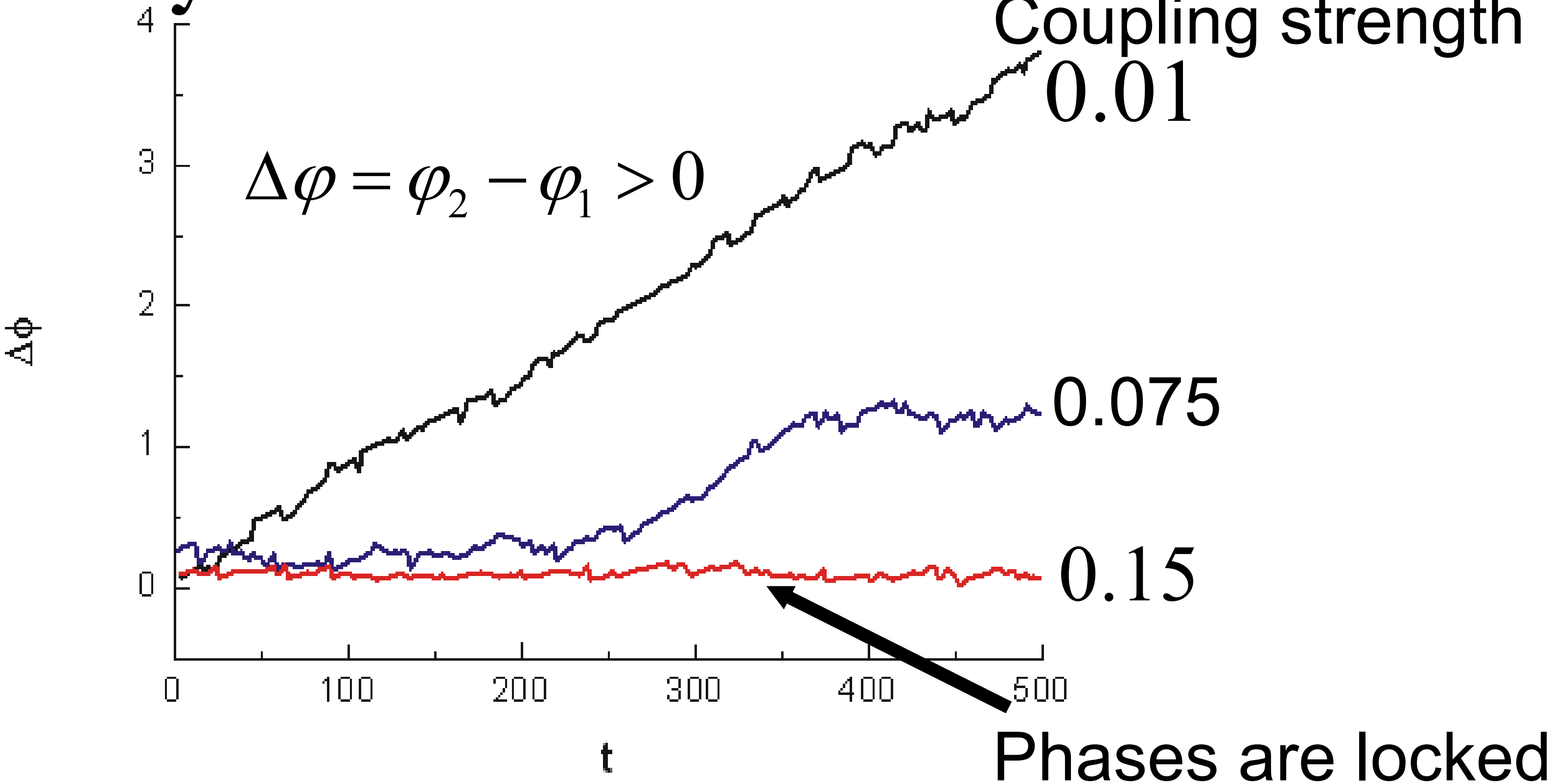




$$\varphi_{1,2} = \arctan \frac{y_{1,2}}{x_{1,2}}, \; A_{1,2} = (x_{1,2}^2 + y_{1,2}^2)^{1/2}.$$

Almost all the time the Rössler system is situated on the plane (x,y) moving along the unstable spiral

Phase dynamics



Phase lead synchronization in unidirectionally coupled chaotic oscillators

Physcon 2011
5th International Scientific Conference on Physics and Control, León (Spain), 2011

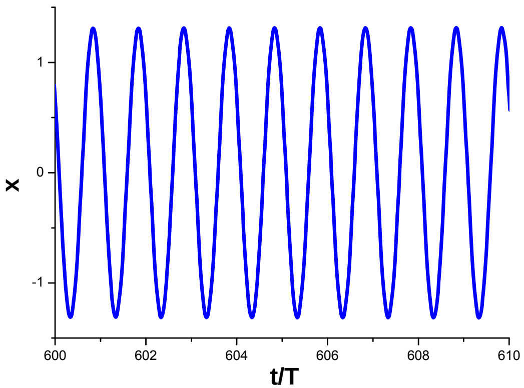
Two unidirectionally coupled Van der Pol oscillators

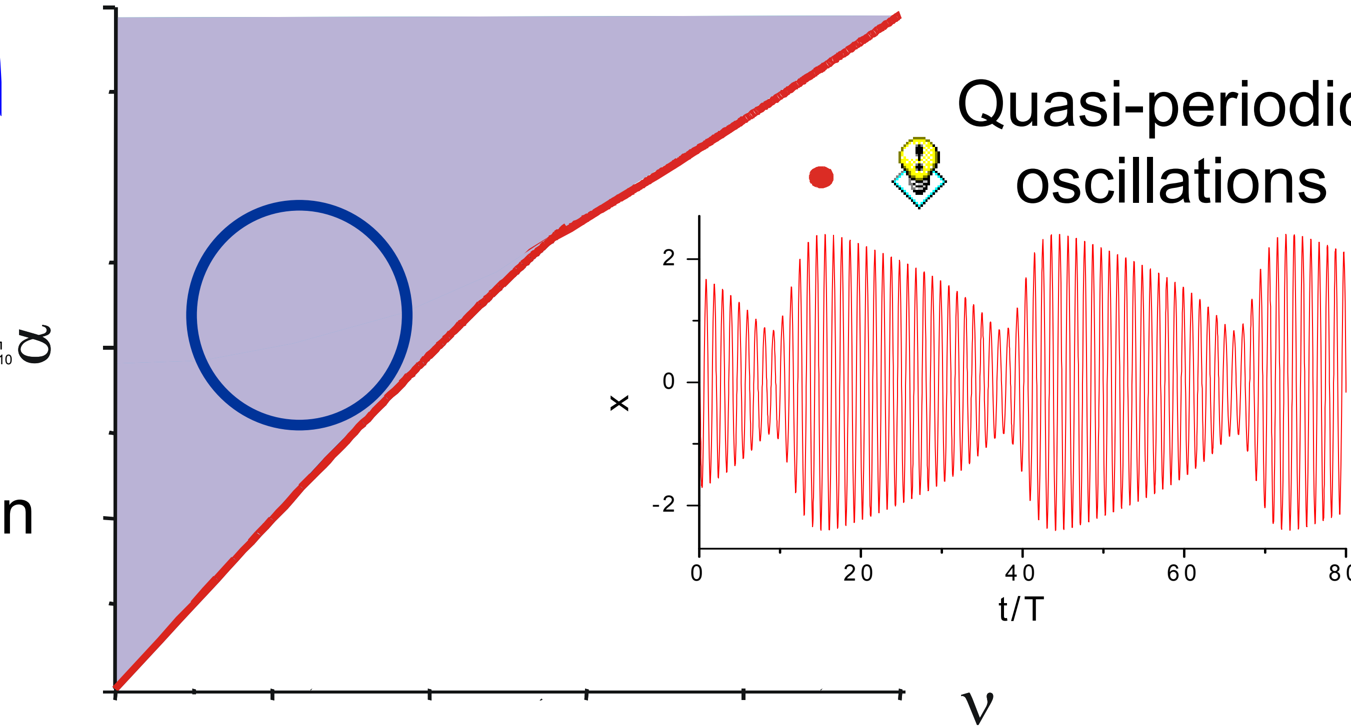
$$\ddot{x}_1 + \omega_{01}^2 x_1 + \varepsilon(x_1^2 - 1)\dot{x}_1 = a\sin(\omega t)$$

$$\ddot{x}_2 + \omega_{02}^2 x_2 + \varepsilon(x_2^2 - 1)\dot{x}_2 = a\sin(\omega t) + k(x_1 - x_2)$$

$$\varepsilon = 0.1, \omega_{01,} = 0.99, \omega_{02} = 1, \omega = 1.046$$

Nonautonomous self-sustained oscillator





Quasi-periodic oscillations

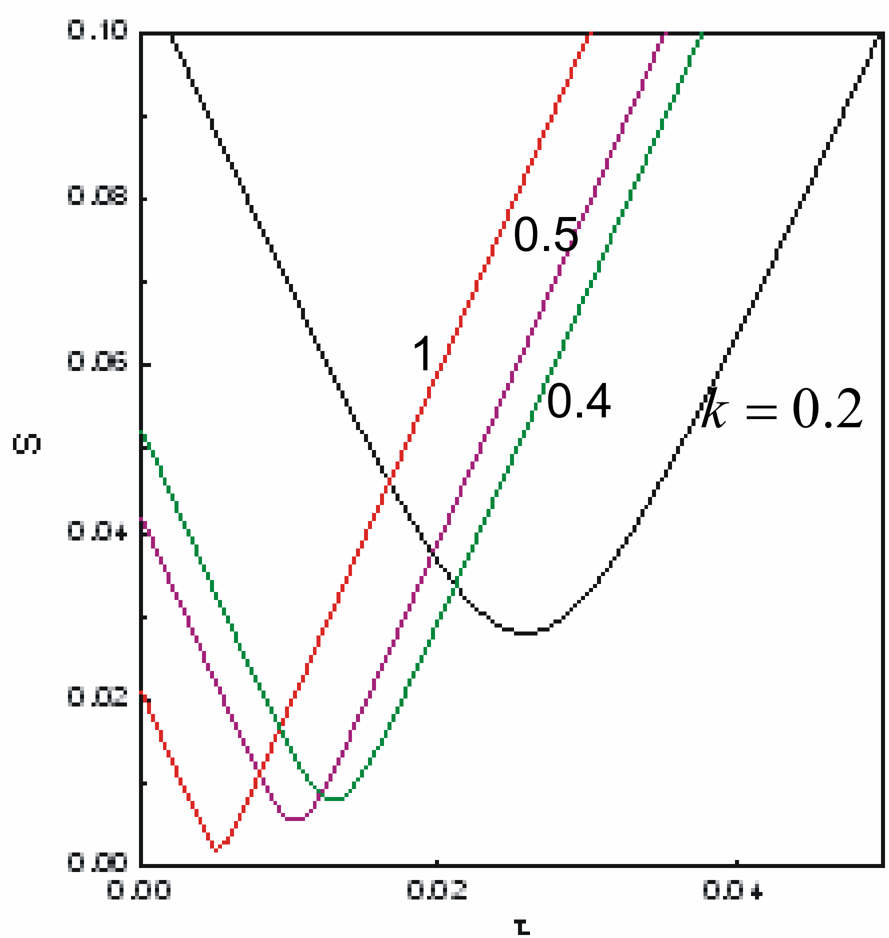
Synchronization regime

$$\alpha = \frac{a}{2\varepsilon\omega}$$ - external force,

$$\nu = \frac{\omega^2 - \omega_0^2}{\varepsilon\omega}$$ - detuning of frequencies

Phys. Lett. A 361, 323 - 331 (2007). Phys. Lett. A 350, 349 - 354 (2006).
Phys. Rev. E 72, 026203 (2005).

Similarity function



2

$$S^2(\tau) = \frac{<[x_2(t) - x_1(t+\tau)]^2>}{[<x_1{}^2><x_2{}^2>]^{1/2}}$$

Phase lead synchronization - the similarly function has a deep minimum at some value of time-shift τ

The further increase of the coupling strength k - anticipating synchronization:

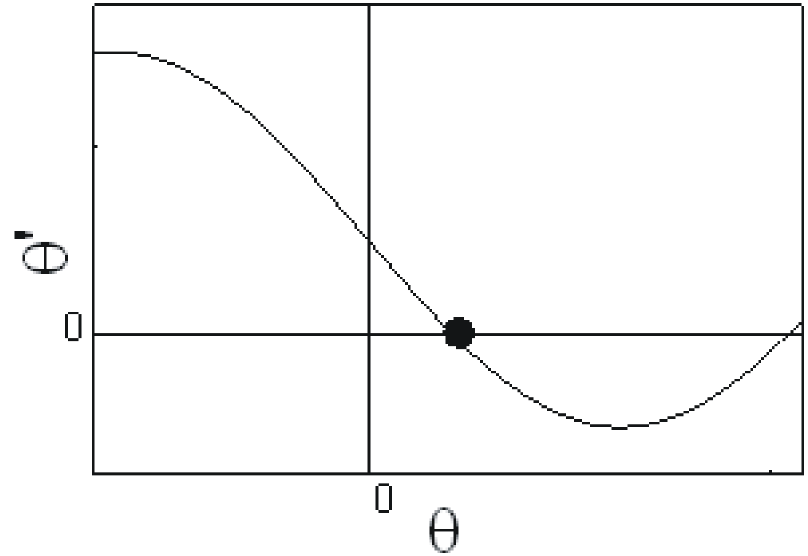
$$x_2(t) = x_1(t+\tau)$$

Analytical treatment

Asymptotical Adler equation

$$2\dot{\theta} = \omega_{01}^2 - \omega_{02}^2 + k\frac{A_1}{A_2}\cos\theta$$

$$\dot{\theta} = 0$$



Positive stable fixed point

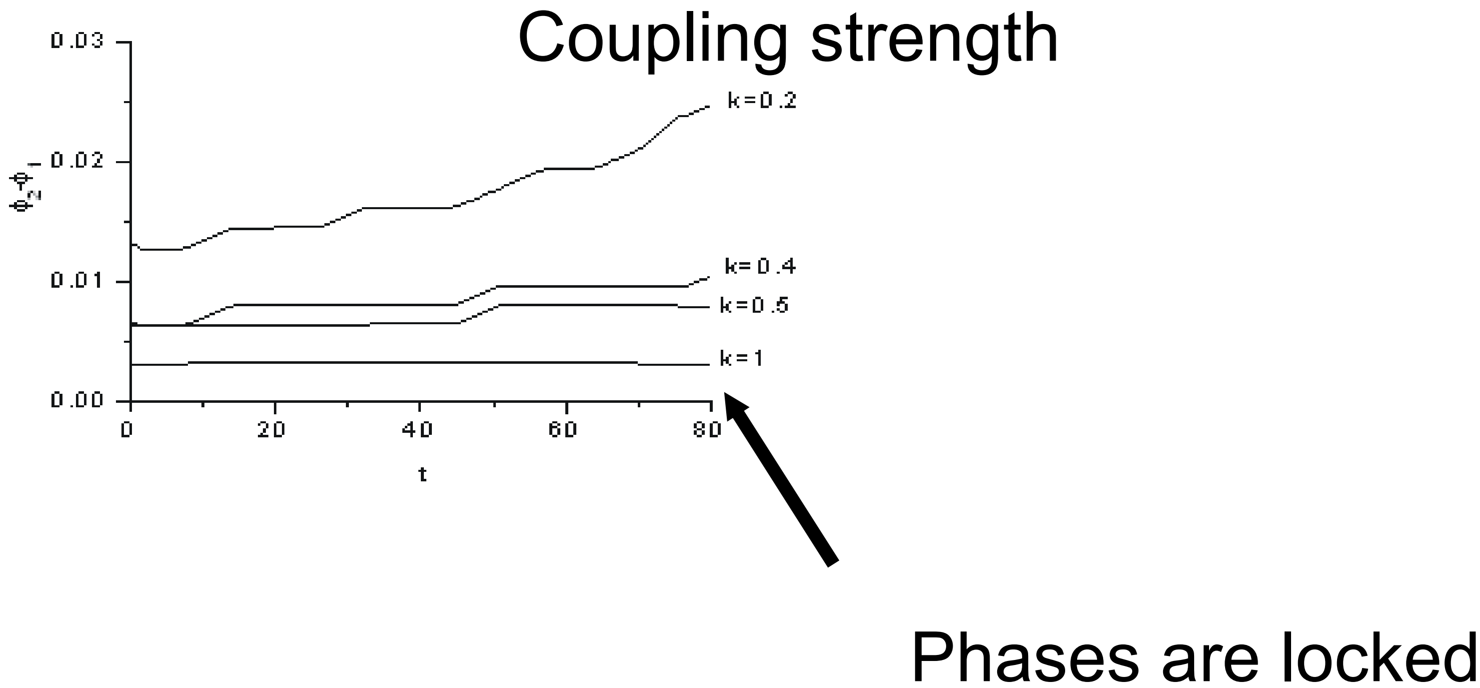
$$\theta = \varphi_2 - \varphi_1$$

$$\theta_0 = \arccos\left[\frac{\omega_{02}^2 - \omega_{01}^2}{k}\right] > 0,$$

$$k > \omega_{02}^2 - \omega_{01}^2$$

$$\omega_2 > \omega_1$$

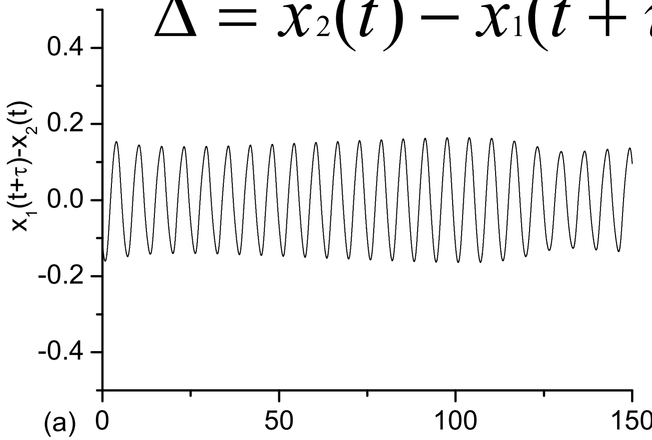
Phase dynamics



We have the phase lead synchronization

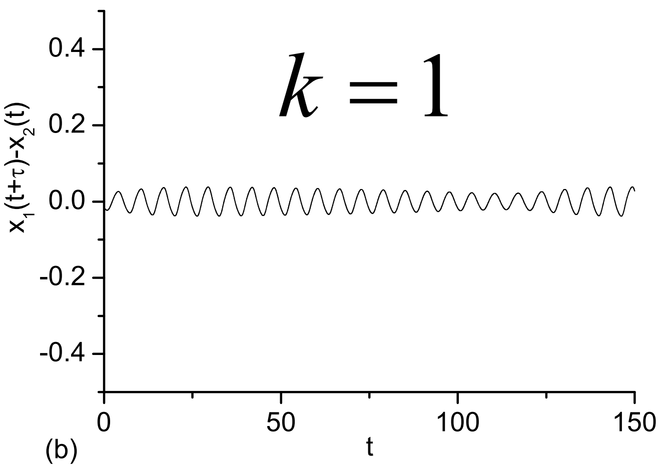
Anticipating synchronization

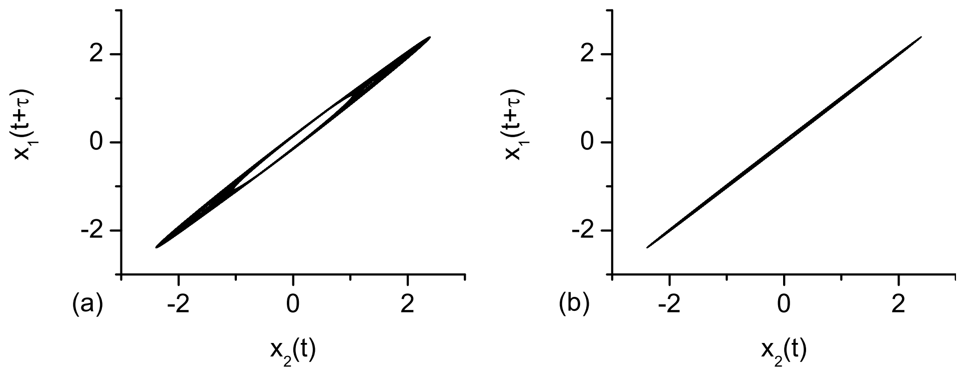
$$\Delta = x_2(t) - x_1(t+\tau)$$



$$k = 0.2$$

$$k = 1$$





Conclusions

- A phenomenon of phase lead synchronization has been detected and analyzed numerically and analytically in unidirectionally coupled nonautonomuos oscillators.
- The phase lead synchronization transforms to the anticipating synchronization for a sufficiently strong coupling strength.
- The anticipating of the dynamics can appear in unidirectionally coupled nonautonomuos oscillators without delay terms in either drive or response systems provided the response system is faster then the drive.

Phase anticipating in two forced self - sustained oscillators

Thank you

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