A twenty-year review of time-delay feedback control and recent developments

Kęstutis Pyragas

Center for Physical Sciences and Technology, Vilnius, Lithuania

FIZINIŲ IR TECHNOLOGIJOS MOKSLŲ CENTRAS

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EUROPOS SĄJUNGA

Kuriame Lietuvos ateitį

Lietuvos mokslo taryba

International Symposium on Nonlinear Theory and its Applications (NOLTA 2012)

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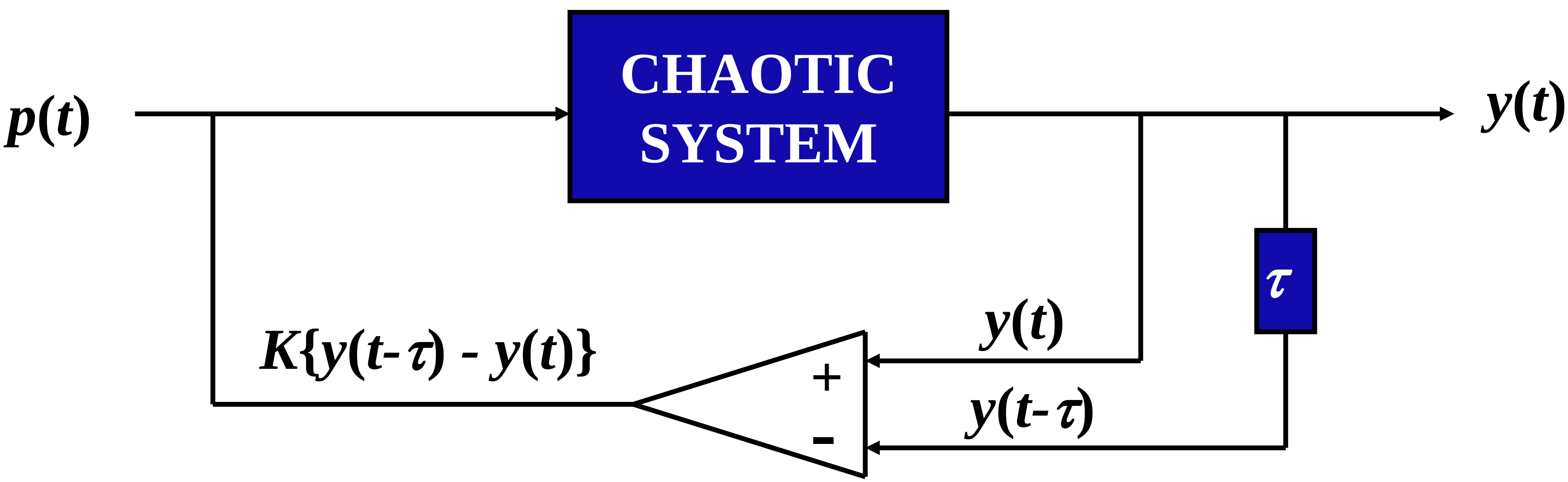
Outline

- Introduction to the delayed feedback control (DFC) algorithm
- Experimental implementations
- Important modification: Extended DFC (EDFC)
- Analytical approaches for DFC systems (linear theory)
- On global properties of the DFC algorithm
- Adaptive modifications
- Odd number limitation (history and recent developments)
- The list of other DFC problems
- Conclusions

Delayed feedback control algorithm

K. Pyragas, Phys. Lett. A 170, 421 (1992)



$$\dot{\vec{x}} = \vec{f}(\vec{x}, p)$$

$$y(t) = g(\vec{x})$$

$$p = p(t) = K[y(t) - y(t-\tau)]$$

τ – period of unstable periodic orbit

1704 citations

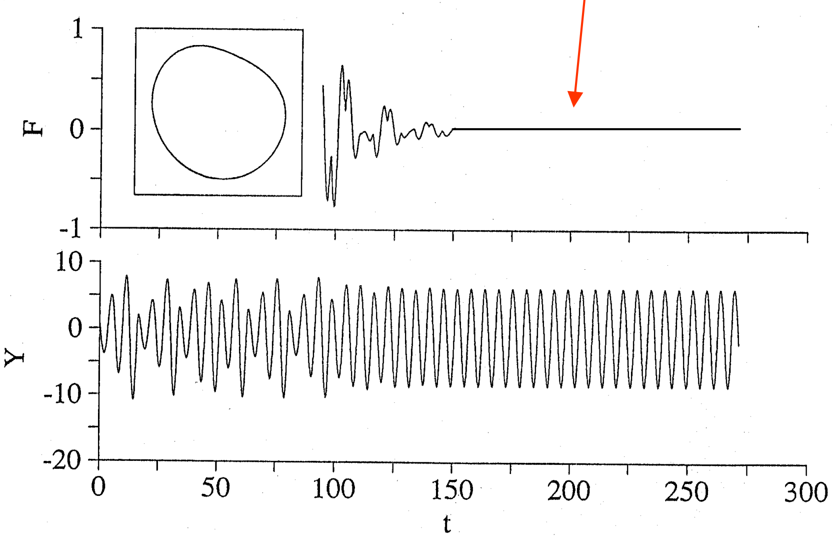
Demonstration for the Roessler system

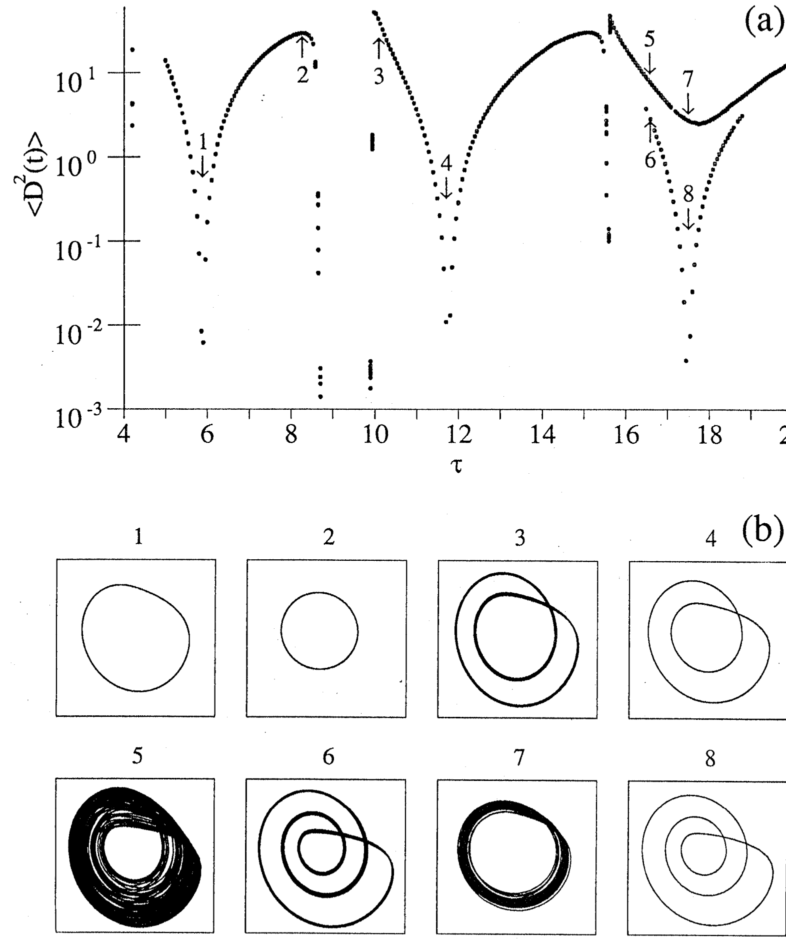
$$\dot{x} = -y - z$$

$$\dot{y} = x + a\,y \;\; + K\{y(t-\tau) - y(t)\}$$

$$\dot{z} = b + z(x - c)$$

The method is noninvasive





- Matlab animation

Experimental implementations (I)

Electronic chaos oscillators

- Pyragas, Tamaševičius (1993)
- Gauthier et al. (1994)
- Kittel et. al. (1994)
- Celka (1994)
- Socolar et al. (1994)
- Benner et al. (1997)
- Sukow et al. (1997)
- Chang et al. (1998)
- Just et al. (1999, 2000)
- Loewenich et al. (2004)
- Boccaletti et al. (2004)
- Choe et al. (2005)
- Ahlborn, Parlitz (2006)
- Hoehne et al. (2007)
- Tamaševičius et al. (2007)
- Loewenich et al. (2010)

Lasers

- Belawski et al. (1994)
- Erneux et al. (1995)
- Lu, Yu, Harrison (1998)
- Arecchi et al. (2002)
- Boccaletti et al. (2004)
- Bielawski et al. (2005)
- Schikora et al. (2006, 2008)
- Dahms et al. (2010)
- Schicora at al. (2011)

Chemical systems

- Schneider et al. (1993)
- Parmananda et al. (1999)
- Tsui, Jones (2000)
- Kiss et al. (2000)
- Kim et al. (2001)
- Bertram et al. (2003)
- Beta et al. (2003)
- Kiss et al. (2006, 2008)

Experimental implementations (II)

Mechanical pendulums
- Hikihara, Kawagoshi (1996)
- Christini et al. (1997)
- Sieber et al. (2008)

Gas discharge systems
- Pierre et al. (1996)
- Mausbach et al. (1997)
- Wei et al. (2004)

Plasma
- Gravier et al. (1999)
- Fukuyama et al. (2002)
- Fukuyama et al. (2006)

Ferromagnetic resonance
- Ye et al. (1995)
- Benner et al. (1997)

DC-DC boost converter
- Natsheh et al. (2009)

Walking control of robots
- Sugimoto, Osuka (2002)

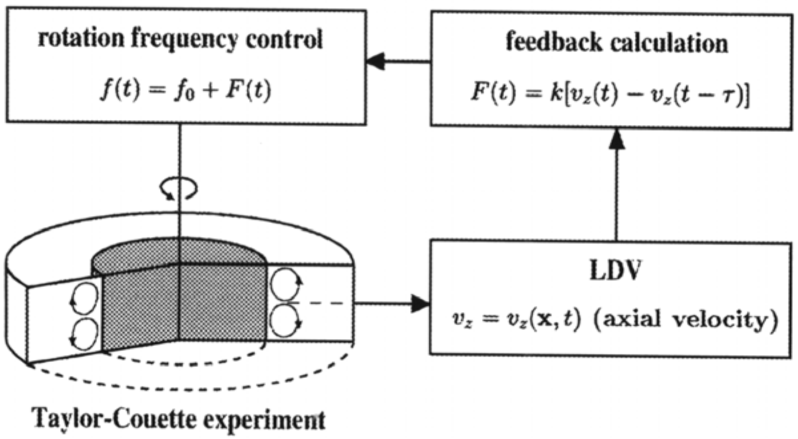
Cardiac systems
- Hall et al. (1997)
- Rappel et al. (1999)
- Berger et al. (2007)

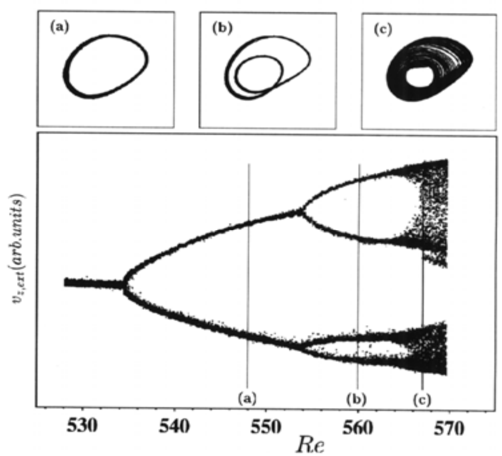
Chaotic Tailor-Couette flow
- Lüthje et al. (2001)

Atomic force microscope
- Yamasue et al. (2009)

Control of chaotic Taylor-Couette flow

Lüthje, Wolf, Pfister, PRL (2001)

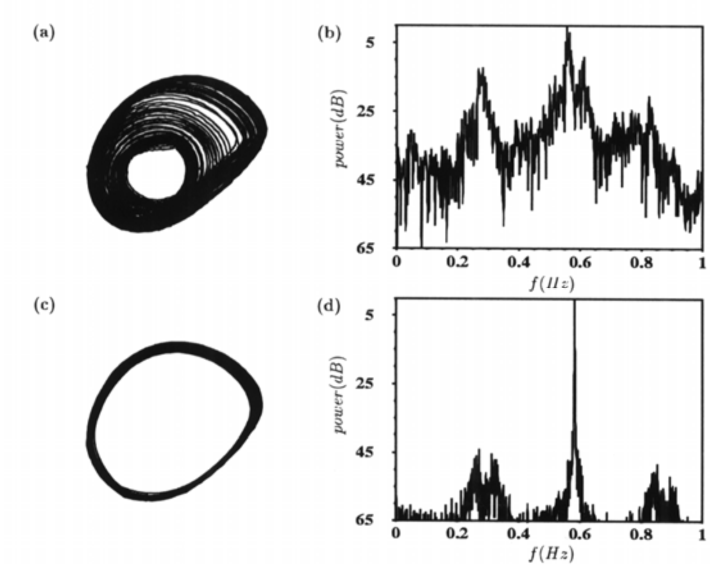




$$\mathrm{Re} = 2\pi f\,(r_\mathrm{o} - r_\mathrm{i})\, r_\mathrm{i} / \nu$$

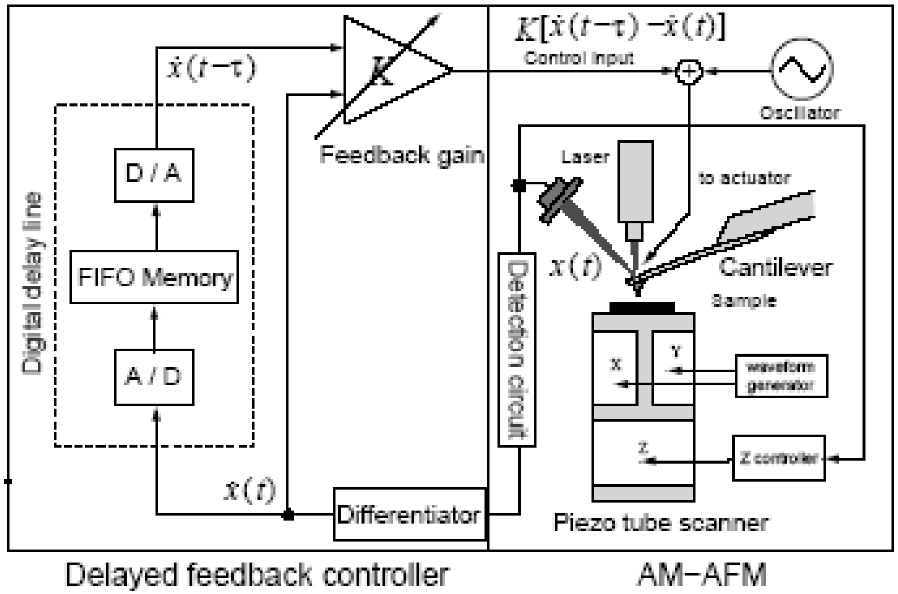
$v_z(\mathbf{x},t)$ - output (axial velocity)

f - control parameter



Controlling chaos in dynamic-mode atomic force microscope

K.Yamasue et al., PLA (2009)



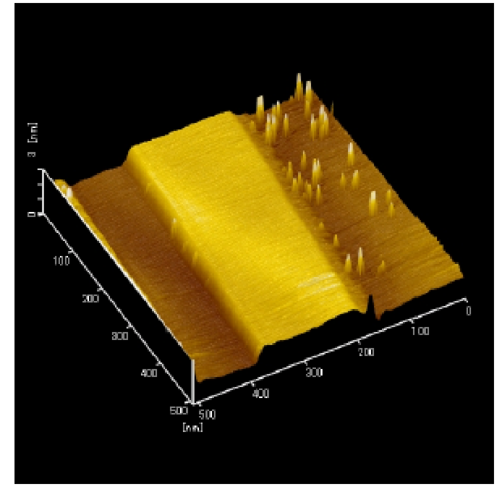
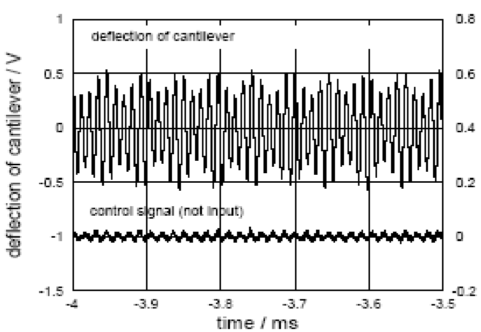
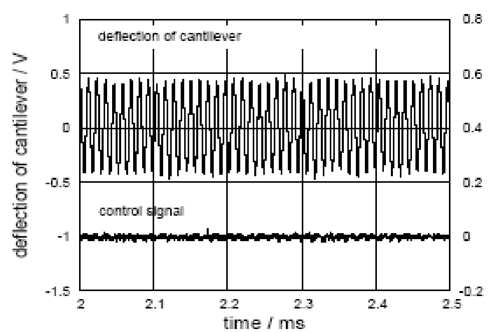


Image without control



without control



with control

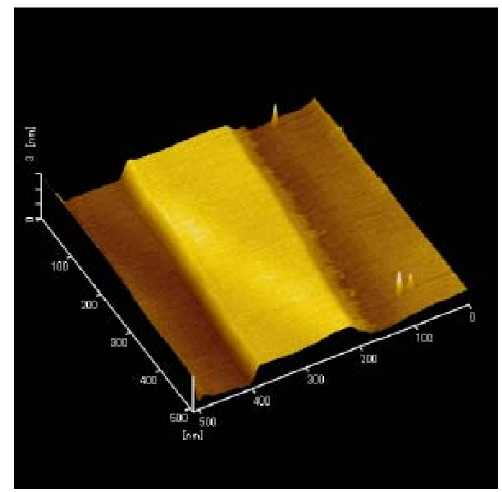


Image with control

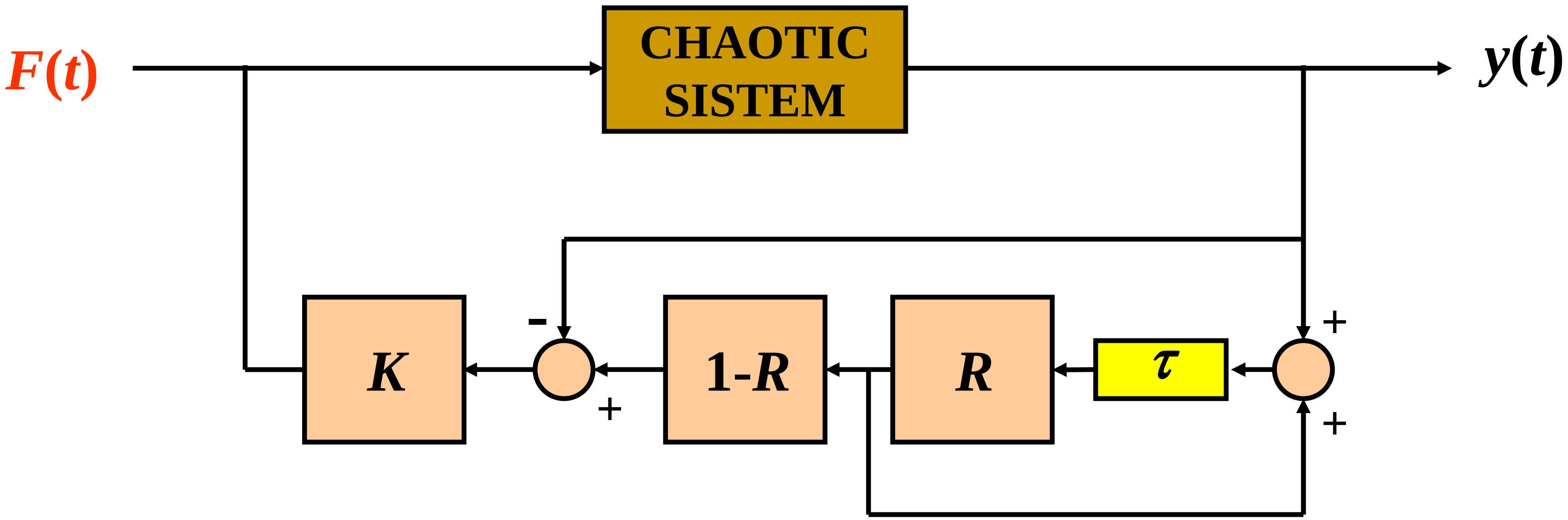
Important modification: Extended DFC (EDFC)

Socolar, Sukov, Gauthier, PRE (1994) **[Pyragas, PLA (1995)]**

$$F(t) = y(t) - y(t-\tau) + R[y(t-\tau) - y(t-2\tau)] + R^2[y(t-2\tau) - y(t-3\tau)] + \cdots$$

$$= y(t) - (1-R)\sum_{k=1}^{\infty} R^{k-1} y(t-k\tau),$$

$|R|<1$ - convergence condition



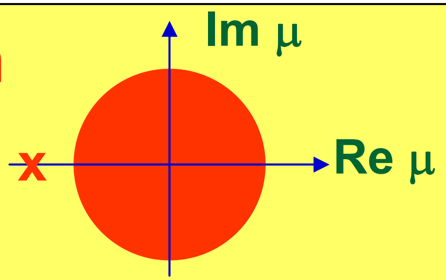
- **Enables the stabilization of highly unstable orbits**
- **Simpe implementation with a single delay line (Fabri-Perrot inerfer.)**

Analytical approaches for DFC systems (linear theory)

- **Close to a period-doubling bifurcation**

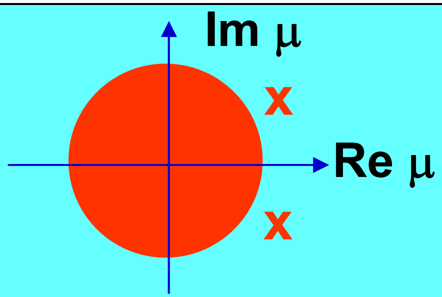
W. Just at al, Phys. Rev. Lett. (1997, 2004)

K. Pyragas, Phys. Rev. E (2002)



- **Close to a Nejmark-Sacker bifurcation**

T. Pyragienė, K. Pyragas, Phys. Rev. E (2005)

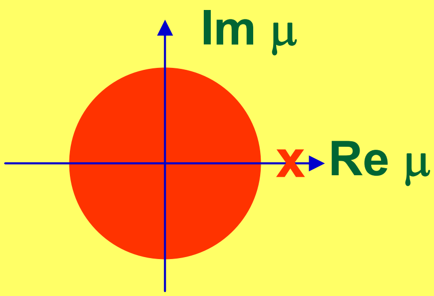


- **Close to a subcritical Hopf bifurcation based on unstable controller**

K. Pyragas, at al, Phys. Rev. E (2004)

V. Pyragas, K. Pyragas, Phys. Rev. E (2006)

K. Hoene et al., Phys. Rev. Lett. (2007)



On global properties (basins of attraction) of the DFC

- The linear analysis is insufficient to guarantee experimental success of DFC algorithm, because the control performance may strongly depend on the basin of attraction of the stabilized state.

Algorithms for improving the global properties:

- Limitation of the size of the control force by a simple cutoff [K. Pyragas, PLA (1992)]

- Two-step DFC algorithm [**Tamaševičius et al., PRE (2007)**]

- Using ergodicity of chaos [K. Pyragas and V. Pyragas, **PRE (2009)**]

Adaptive DFC algorithms

- The period of UPO (delay time) and control gain are not known a priory. In experiments, an on line adaptive tuning of these parameters is desired.

Adaptive tuning of the delay time:

- **A. Kittel, J. Parisi, and K. Pyragas, PLA (1995)**
- **H. Nakajima, H. Ito, Y. Ueda, IEICE Trans. Comput. Sci. (1997)**
- **G. Chen, X. Yu, IEEE Trans. Circ. Syst. I (1999)**
- **X. Yu, IEEE Trans. Circ. Syst. I (1999)**
- **G. Herrmann, PLA (2001)**
- **I.Z. Kiss, Z. Kazsu, V. Gáspár, Chaos (2006)**
- **K. Pyragas and V. Pyragas, PLA (2011)**

Adaptive tuning of the control gain:

- **J. Lehnert, et al., Chaos (2011)**

Adaptive tuning of both the delay time and control gain:

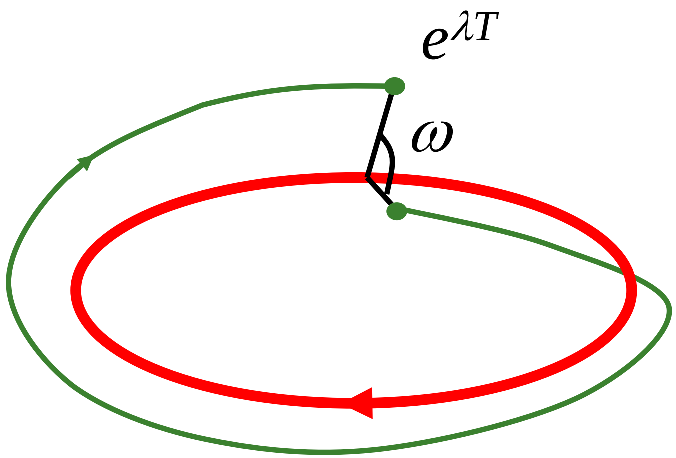
- **W. Lin, et al., PRE (2010);**

Odd number limitation (ONL)

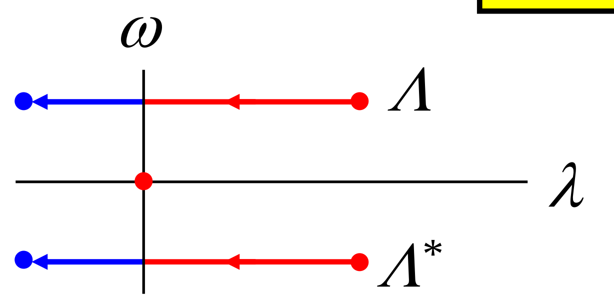
Ushio (1996), Nakajima and Ueda (1997,1998), Just at al. (1997)

Limitation: The DFC method fails for any periodic orbits with an odd number of real Floquet multipliers >1

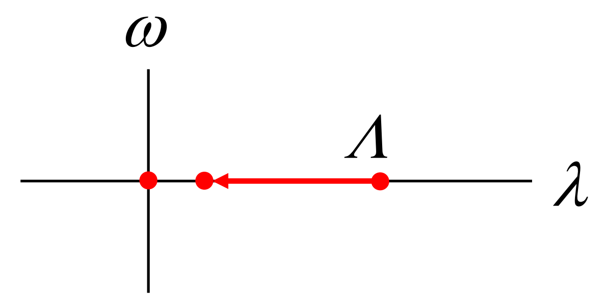
FE: $\Lambda = \lambda + i\omega$ **FM:** $\mu = e^{\Lambda T}$



The method works for $\omega \neq 0$



The method fails for $\omega = 0$



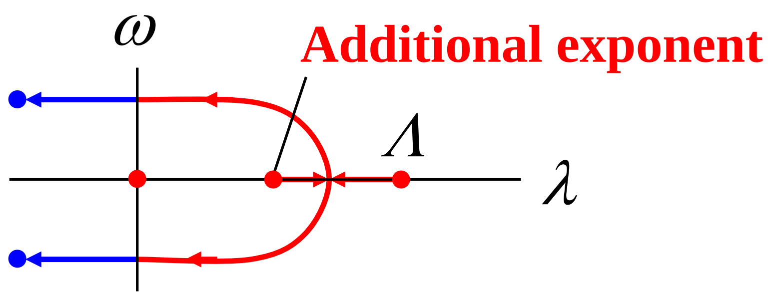
B. Fiedler at al., PRL (2007) → **Refuting for autonomous systems**

Unstable delayed feedback controller

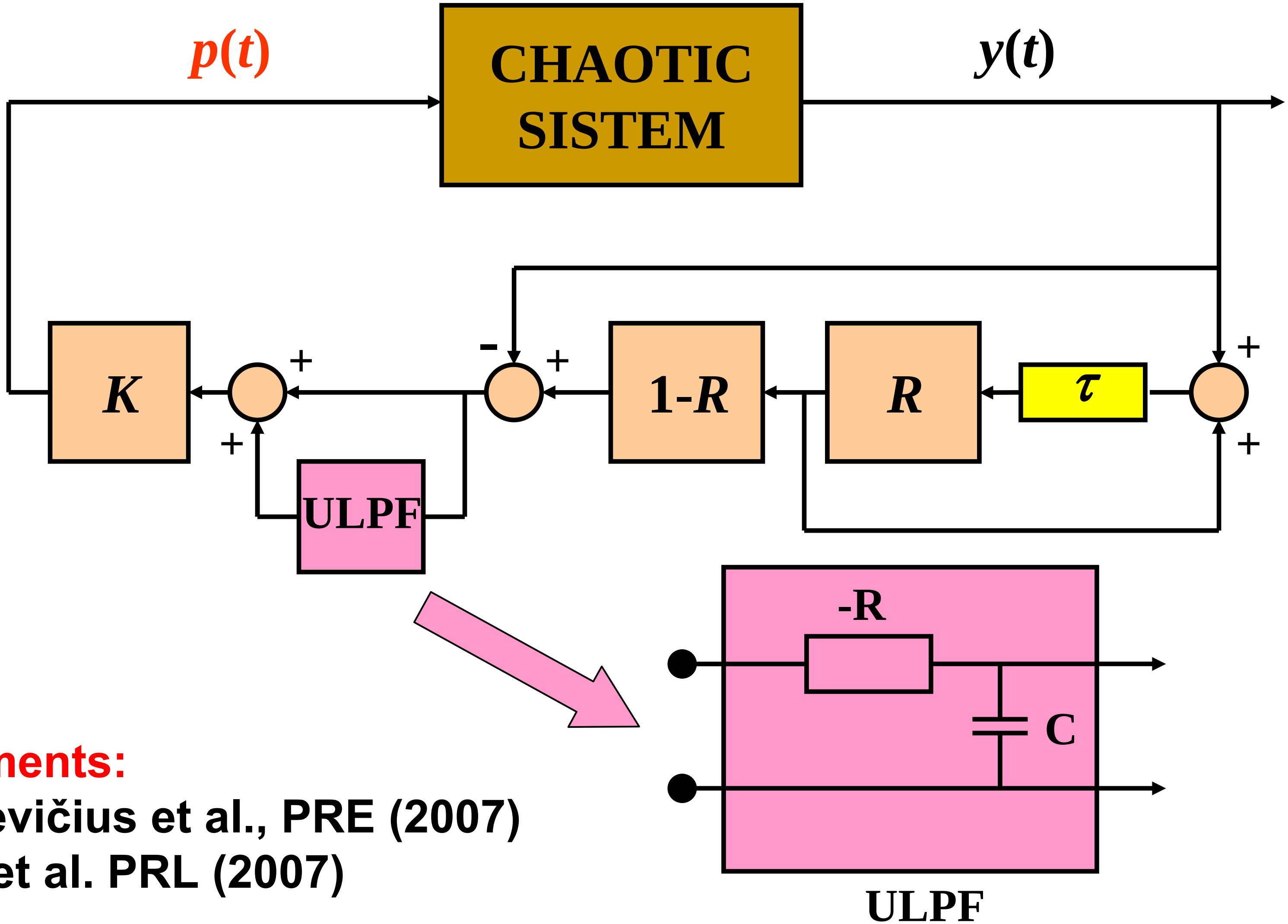
K. Pyragas, Phys. Rev. Lett. 86, 2265 (2001)

Counterintuitive idea:

Artificially enlarge a set of real Floquet multipliers greater than unity to an even number by introducing into a feedback loop an unstable degree of freedom



Block diagram of the unstable EDFC (UEDFC)



Experiments:

Tamaševičius et al., PRE (2007)

Hoene et al. PRL (2007)

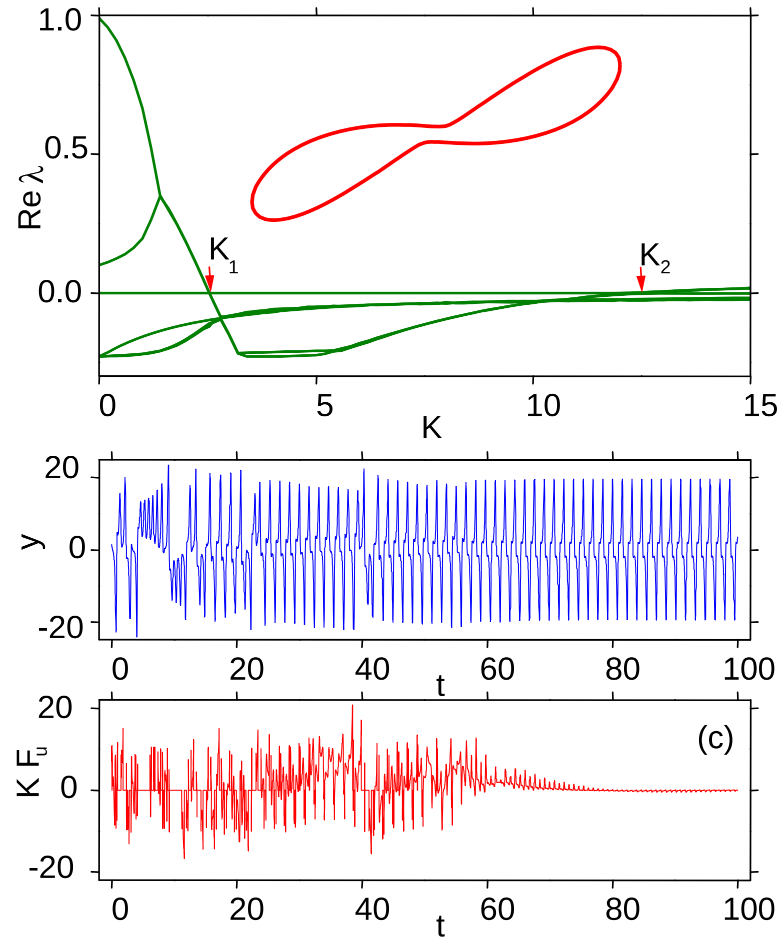
Demonstration for the Lorenz system

$$\dot{x} = -\sigma(y+z)$$
$$\dot{y} = rx - y - xz - KF_u(t)$$
$$\dot{z} = xy - bz$$

UEDFC:

$$F_u(t) = F(t) + w(t)$$
$$F(t) = y(t) - y(t-\tau) + R\,F(t-\tau)$$
$$\dot{w} = \lambda_c^0 w(t) + \left(\lambda_c^0 - \lambda_c^\infty\right) F(t)$$

$$R < 1, \quad \lambda_c^0 > 0$$



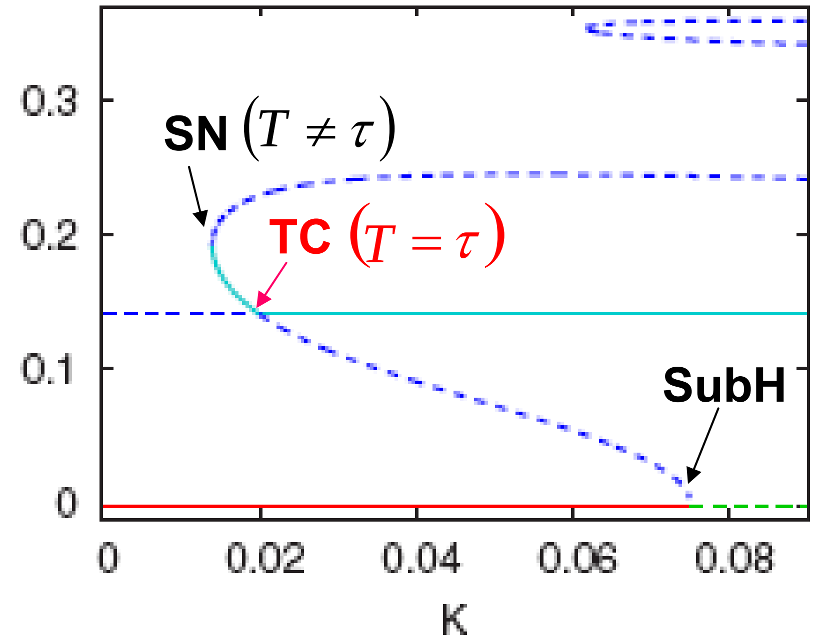
The odd number limitation revisited

Fiedler at al., PRL (2007) → **Counterexample**

Normal form of SubH bifurcation under DFC:

$$\dot{z}(t)=\left[(\lambda+i)+(1+i\gamma)|z(t)|^2\right]z(t)-Ke^{-i\beta}\left[z(t)-z(t-\tau)\right]$$

Complex gain (rotaiting feedback)



- **DFC induces a pair of orbits (a saddle-node bifurcation) with** $T\neq\tau$

- **The UPO coalisce with iduced orbit at a transcritical bifurcation and exchange the stability**

Corrected criterion for the odd number limitation

A. Amann and E.W. Hooton, PRL (2012)

An UPO of autonomous sysytem cannot be stabilized by DFC if:

$$(-1)^m \lim_{\tau \to T} \frac{\tau - T}{\tau - \Theta(\tau)} < 0$$

m - the number of real Floquet multipliers larger than 1

T - the period of UPO

τ - slightly mismatched delay time

$\Theta(\tau)$ - the period of the induced orbit at the mismatched delay

Analytical computation of the period $\Theta(\tau)$

Phase reduction theory approach

Novičenko, Pyragas, Physica D (2012) → **Extension of phase reduction theory to the systems with time delay**

Novičenko, Pyragas, PRE (2012) → **Application of phase reduction theory to the EDFC systems**

EDFC at a small time delay mismatch:

$$\dot{\vec{x}} = \vec{f}(\vec{x}(t), u(t)), \quad \tau \neq T$$
$$u(t) = g(x(t))$$
$$u(t) = K\left[(1-R)\sum_{j=1}^{\infty} R^j s(t - j\tau) - s(t)\right]$$

→ **Phase reduced equation:**

$$\dot{\varphi} = 1 + (\tau - T)\vec{z}^T(\varphi)\vec{\psi}(t) + O\left((\tau - T)^2\right)$$

Phase response curve (PRC) (for $\vec{z}$)

Analytical computation of the period $\Theta(\tau)$

The profile of the PRC is independent of K and R !

Only the amplitude of the PRC depends on K and R

The PRC of the controlled system for any K and R can be expressed trough the PRC of the control-free UPO (basic PRC):

Eqs. for the PRC of control-free UPO:

$$\dot{\vec{\rho}}^T(t) = -\vec{\rho}^T(t) A_0(t)$$

$$\vec{\rho}^T(0)\vec{\xi}(0) = 1$$

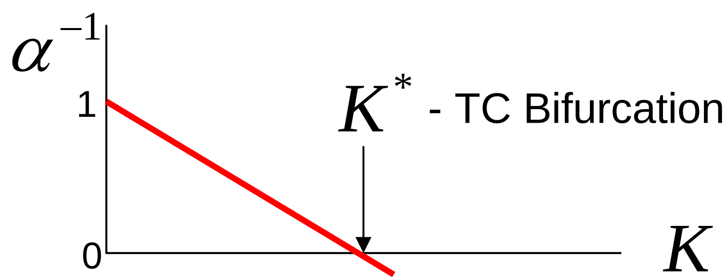
PRC in the presence of EDFC:

$$\vec{z}(t) = \alpha(K,R)\vec{\rho}(t)$$

$$\Theta(\tau) = T - (\tau - T)[\alpha(K,R) - 1] + O\left((\tau - T)^2\right)$$

The Amann and Hooton criterion in terms of the PRC theory:

$$(-1)^m \alpha^{-1}(K,R) < 0$$



The list of other DFC problems not discussed in this review

- **Control of oscillation coherence in noisy systems**
  - Goldobin, Rosenblum, Pikovsky, Physica A (2003); PRE (2003);
  - Janson, Balanov, Schoell, PRL (2004);
  - Balanov, Janson, Schoell, Physica D (2004);
  - Hauschildt, Janson, Balanov, Schoell. PRE (2006);
  - Pomplun, Amann, Schoell, EPL (2005) ; PRE (2007);
- **Control of synchronization in neural networks**
  - Rosenblum, Pikovsky, PRL (2004); PRE (2004);
  - Hauptmann, Popovych, Tass, Neurocomputing (2005); Biol. Cybern. (2005);
  - Popovych, Hauptmann, Tass, PRL (2005); Biol. Cybern. (2006); Int. J. Bif. Chaos (2006;
  - Popovych, Hauptmann, Tass, PRE (2010);
- **Control of spatio-temporal patterns in reaction-diffusion systems**
  - Baba, Amann, Schoell, Just PRL (2002);
  - Beck, Amann, Schoell, Socolar, Just, PRE (2002);
  - Balanov, Beato, Janson, Engel, Schoell PRE (2006);
  - Schneider, Schoell, Dahlem, Chaos (2009);
- **DFC based bifurcation analysis for experiments**
  - Sieber, Gonzalez-Buelga, Neild, Wagg, Krauskopf, PRL (2008);
- **DFC with the variable and distributed time delay**
  - Gjurchinovski, Urumov, EPL (2008);
  - Juengling, Gjurchinovski, Urumov, http://arxiv.org/pdf/1202.0519.pdf (2012)

Conclusions

The delayed feedback control is still an active area of research

Hopefully, new interesting theoretical and experimental results will appear in the near future

Acknowledgments

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