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15 May 1995

PHYSICS LETTERS A

Physics Letters A 201 (1995) 42–46

An electronic analog of the Mackey–Glass system

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Received 8 December 1994; revised manuscript received 9 February 1995; accepted for publication 9 March 1995
Communicated by A.P. Fordy

Abstract

An electronic oscillator which simulates the Mackey–Glass evolution equation is designed and investigated experimentally. The circuit contains a tunable delay unit, a nonlinear device, and an *RC* filter. It provides an easy way to generate high-dimensional signals. The largest estimated correlation dimension for the chaotic oscillations ranges up to ≈ 7.

1. Introduction

Higher-dimensional ($D \geqslant 4$) dynamical systems can exhibit very complex hyperchaotic behaviour. An example of an infinite-dimensional ($D \to \infty$) system provides the Mackey–Glass (MG) mathematical model for haematologic disorders [1]. The model is described by a delay differential equation (DDE) usually given in the following form,

$$\dot{x} = \frac{a x_\tau}{1 + x_\tau^n} - bx, \quad x_\tau \equiv x(t - \tau). \tag{1}$$

Most investigations on the MG system deal with $a = 0.2$, $b = 0.1$, and $n = 10$. The state of the system strongly depends on the delay time τ. Chaotic oscillations at $\tau \geqslant \tau_c = 16.8$ are preceded by a stable steady state and successively doubled periodic orbits. The MG model has been studied numerically by Farmer [2], also by Grassberger and Procaccia [3] to obtain Lyapunov exponents, entropies and various types of dimensions.

Eq. (1) can be reduced by dividing the right-hand-side terms by b and scaling the time quantities $t \to bt$, $\tau \to b\tau$ (evidently, $\tau_c = 16.8$ becomes $\tau_c \to b\tau_c = 1.68$):

$$\dot{x} = \frac{2 x_\tau}{1 + x_\tau^{10}} - x. \tag{2}$$

In this Letter we report on an analog electronic circuit, which simulates the mathematical MG model, that is, Eq. (2). We note that an analog device has been employed by Losson, Mackey and Longtin [4] to simulate a DDE. However, the DDE studied by analog means in Ref. [4] is a piecewise constant model, which is different from the MG equation. Moreover, the analog instrument used in Ref. [4] is an analog computer. As usual it contains an integrator, a summator, one or more inverting amplifiers in addition to a delay unit and a nonlinear transformer. The difference between an analog computer, which turned out to be rather convenient tool to explore various models for chaos [4], and the "intrinsic" electronic circuits exhibiting chaotic behaviour has been emphasized in a review paper by Matsumoto [5].

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SSDI 0375-9601(95)00208-1