Physics Letters A 180 (1993) 99–102
North-Holland

Experimental control of chaos by delayed self-controlling feedback

K. Pyragas [1,2]
Physical Institute, University of Tübingen, W-7400 Tübingen, Germany

and

A. Tamaševičius
Semiconductor Physics Institute, 2600 Vilnius, Lithuania

Received 14 April 1993; revised manuscript received 11 June 1993; accepted for publication 25 June 1993
Communicated by A.P. Fordy

A recently proposed method [K. Pyragas, Phys. Lett. A 170 (1992) 421] of chaos control by a small time-continuous perturbation is realized experimentally. The required delayed feedback is achieved by a specially designed analogue circuit using a simple delay line. The experimental results are supplemented by computer simulation.

Chaotic attractors typically have embedded within them an infinite number of unstable periodic orbits. A few years ago Ott, Grebogi and Yorke (OGY) employing this key observation suggested a general way to control chaos [1]. A desired periodic motion of an originally chaotic dynamic system can be achieved through only small, carefully chosen temporal perturbations made in an accessible system parameter. This idea has been developed [2–4] and applied to a variety of dynamical systems, including mechanical, electronic, optical systems [5–8]. The OGY method implies, as a rule, a computer analysis of the system. Moreover, it deals with the Poincaré maps, so the corresponding parameter perturbations appear to be time-discrete. This leads to some restrictions in applying the method to experimental systems in which small random noise can result in occasional bursts where the trajectory wanders far from the controlled periodic orbits [1].

Recently one of us (K.P.) proposed two new methods to control chaos with small perturbations involved as a time-continuous feedback [9]. One of these methods is particularly convenient for experimental applications. The method is based on the feedback perturbation constructed in the form of the difference between the delayed output signal and the output signal itself. This perturbation keeps the desired unstable periodic orbit unperturbed if the delay time of the output signal coincides with its period. Stabilization is achieved by adjusting the weight of the perturbation. The feedback is self-controlled and the method does not require a computer analysis of the state of the system. Unlike the OGY method, it is robust to noise. The continuous feedback perturbation holds the system on the desired periodic orbit even in the presence of sufficiently large noise. The system does not experience any occasional bursts into the region far from the periodic orbit. The increase in noise leads only to the self-increase of the feedback perturbation and to smearing out of the periodic orbit.

The method can be carried out by a simple analogue technique. In ref. [9] it was tested numerically for various chaotic models including autonomous and periodically driven systems. In the present paper we report on the first experimental realization of this method.

The experimental system was an externally driven

[1] Alexander von Humboldt Fellow.
[2] On leave from Semiconductor Physics Institute, Goštauto 11, 2600 Vilnius, Lithuania.

nonlinear oscillator [10,11] (fig. 1), including a tunnel diode, as a negative resistance device. The parameters of the circuit were as follows: $L=17.4$ µH, $C=510$ pF, $R=5.1$ kΩ. The external drive frequency f_{ex} was 3.8 MHz, the drive amplitude A was varied up to 3 V, the dc bias U was always kept less (in contrast to ref. [10]) than the critical value for the appearance of self-sustained oscillations (≈ 70 mV). The employed germanium tunnel diode is characterized by the peak current $I_p=1.5$ mA, and the peak voltage $U_p=67$ mV.

The dynamics of the oscillator is determined by the rate equations

$$\frac{dx}{dt}=-y+c\,,$$

$$\frac{dy}{dt}=x-bN(y)-dy+a\sin\omega t+F(y,\tau)\,, \qquad (1)$$

with $F(y,\tau)\equiv 0$ for the uncontrolled oscillator. The dimensionless variables and parameters are given by

$$x=ZI/U_p\,,\quad y=U/U_p\,,\quad t\Leftarrow t/T_0\,,\quad F=ZI_c/U_p\,,$$
$$Z=\sqrt{L/C}\,,\quad T_0=\sqrt{LC}\,,\quad a=ZA/RU_p\,,$$
$$b=ZI_p/U_p\,,\quad c=U_0/U_p\,,\quad d=Z/R_p\,,$$
$$\omega=2\pi f_{ex}T_0\,,\quad T=2\pi/\omega\,. \qquad (2)$$

Here U is the voltage across the capacitance, I is the current through the inductance, I_c is the control current, $R_p=RR_c/(R+R_c)$ is the resistance of the parallel connected resistances R and R_c, R_c is the load resistance, e.g. due to the control circuit. The normalized current–voltage characteristic of the tunnel diode can be given in the following form,

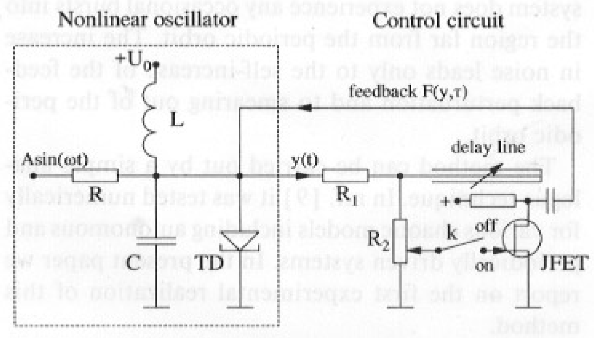

Fig. 1. Experimental setup.

$$N(y)=\frac{y^{\alpha}\exp[\alpha(1-y)]+\beta[\exp(y)-1]}{1+\beta[\exp(1)-1]}\,, \qquad (3)$$

with $\alpha=1.7$ and $\beta=0.001$.

To realize the control electronically a special analogue circuit (fig. 1) was designed. The parameters were as follows: $R_1=1.8$ kΩ, $R_2=260$ Ω, the wave resistance of the delay line $Z_d=260$ Ω ($R_2=Z_d$). The maximum delay time of the variable spiral line was 600 ns (one way), i.e. 1200 ns (double way in the short-circuited line). The transconductance of the transistor $S=11$ mA/V. This circuit can be shown to branch off from the above described oscillator the feedback control current $F(y,\tau)$,

$$F(y,\tau)=K[y(t-\tau)-y(t)]\,,$$

$$K=hpSZ\,,\quad \tau=T_d/T_0\,, \qquad (4)$$

i.e. just in the form as required in the method of ref. [9]. Here $p=R_2Z_d/(R_1R_2+R_1Z_d+R_2Z_d)$, and h is an additional division coefficient ($h\leqslant 1$) due to the slide divider R_2. The T_d is the adjustable double way delay time.

Controlling the unstable periodic orbits is very easy. The delay time T_d is adjusted close to the period of the desired orbit by means of the variable delay line. For the described driven oscillator T_d must be close to m/f_{ex}, where m is the periodicity number of the locked orbit ($m=1$, 2, ...). Then the switch "k" (fig. 1) is turned on and the feedback coefficient K is increased by means of the slide divider R_2 until the stabilization occurs. If necessary the delay time T_d can be corrected and the whole adjustment procedure can be repeated to minimize the control signal.

The experimental results, obtained for two different amplitudes of the external driving voltage applied to the oscillator, are shown in fig. 2, The chaotic states of the uncontrolled oscillator are characterized by the broadband continuous spectra. Meanwhile, there are sharp peaks in the power spectra when the control is on. The position of these discrete lines depends on the periodicity of the stabilized orbits with the lowest frequency corresponding to f_{ex}/m. In the first case (figs. 2b, 2c) stabilization of the orbits with $m=3$ and $m=1$ is evident, while the second case (figs. 2e, 2f) illustrates the control of the cycles with $m=4$ and $m=2$.

Locked into a periodic state and adjusted, the con-

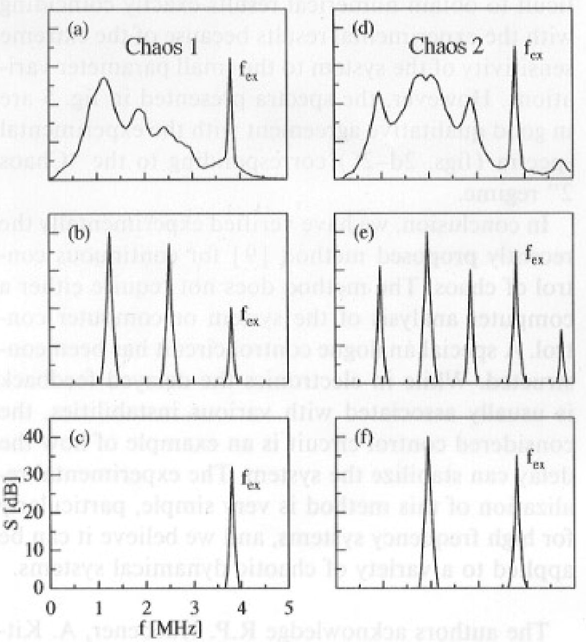

Fig. 2. Experimental power spectra for two different drive amplitudes. (a) Chaotic spectrum, (b) controlled period-three spectrum and (c) controlled period-one spectrum for $A=1.2$ V. (d) Chaotic spectrum, (e) controlled period-four spectrum and (f) controlled period-two spectrum for $A=2.5$ V. All spectra are shown in the basic frequency range $[0, f_{ex}]$. The form of the chaotic spectra at higher frequencies $[f_{ex}, 2f_{ex}]$, $[2f_{ex}, 3f_{ex}]$, ... replicate the spectra in the basic range with an average level decay of 15—20 dB in each of these intervals.

trol signals were extremely small, $I_c=2$–3 µA, or $F=(6$–$8)\times10^{-3}$, corresponding to currents in the oscillator itself of less than 1%.

In addition, some computer calculations were performed. Numerical solutions of eqs. (1) with parameters close to the experimental values ($a=1.40$, $b=4.10$, $c=0.92$, $d=0.15$, $\omega=2.25$) corresponding to the "Chaos 2" regime are presented in figs. 3–5. Figure 3 illustrates the dynamics of the perturbation and the output signal before and after switching on the control. The delay time τ is chosen to be equal to the period of the unstable period-two orbit. The uncontrolled system is characterized by a chaotic behavior with desynchronized signals $y(t)$ and $y(t-\tau)$. Switching on the control results in periodical oscillations corresponding to an initially unstable periodic orbit. The signal $y(t)$ tends to $y(t-\tau)$ and the perturbation F tends to zero. The finite values of F

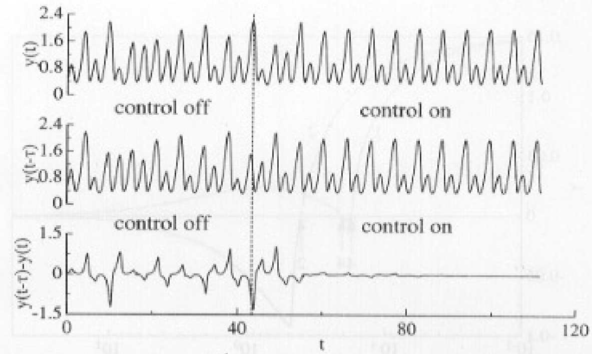

Fig. 3. Calculated dynamics of the output signal $y(t)$, delayed output signal $y(t-\tau)$ and the difference $y(t-\tau)-y(t)$ before and after switching on the control. The vertical dashed line shows the moment of switching on the control. $\tau=2T$, $K=0.5$.

observed experimentally are conditioned by the ohmic losses that are perceptible ($\approx10\%$) in the long spiral delay line.

The stabilization of unstable periodic orbits has been achieved in experiment only in definite intervals of the weight K of the feedback. Theoretically these intervals can be determined from variational equations, defining the Lyapunov exponents of corresponding orbits:

$$\frac{d\delta x}{dt}=-\delta y(t)\,,$$

$$\frac{d\delta y}{dt}=\delta x-\left(b\left.\frac{dN(y)}{dy}\right|_{y=y_m(t)}+d\right)\delta y(t)$$
$$+K[\delta y(t-\tau)-\delta y(t)]\,. \qquad (5)$$

Here $\delta x=x-x_m(t)$, $\delta y=y-y_m(t)$ are the small deviations from the unstable period-m orbit $\{x_m(t), y_m(t)\}=\{x_m(t+mT), y_m(t+mT)\}$. The dependence of the maximal Lyapunov exponents λ on K for the period-two and period-four orbitals are presented in fig. 4. The negative value of $\lambda(K)<0$ define the intervals of K corresponding to the stabilization. As can be seen from the figure, the period-four orbit has only a small stabilization interval, while the interval for the period-two orbit is infinite, i.e. the stabilization of this orbit can be achieved at any $K>K_{min}$, where K_{min} is the threshold of the stabilization defined by $\lambda(K_{min})=0$.

The numerical results of the stabilization at the fixed values τ and K are presented in fig. 5. It is dif-

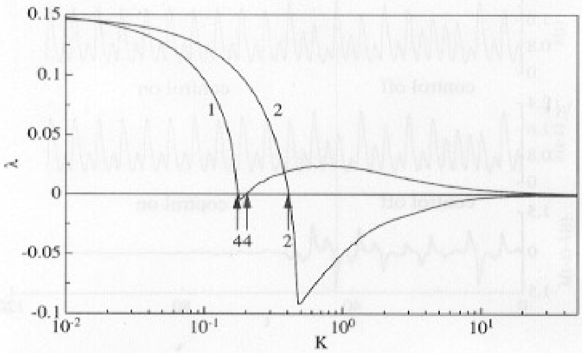

Fig. 4. The dependence of the maximal Lyapunov exponent λ on the weight K of the perturbation for (1) period-four and (2) period-two orbits. The arrows, marked by numbers corresponding to the periodicity of the orbits, show the thresholds of synchronization.

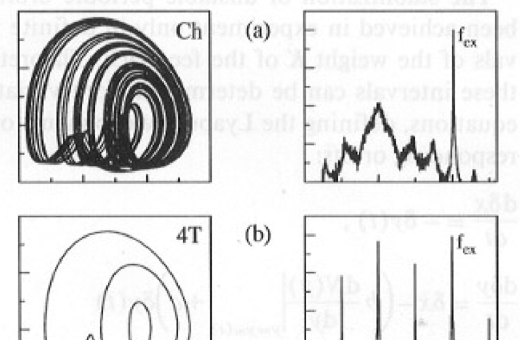

Fig. 5. Calculated phase portraits and power spectra of (a) chaotic oscillations, (b) controlled period-four orbit, $\tau=4T$, $K==0.17$, and (c) controlled period-two orbit, $\tau=2T$, $K=0.5$.

ficult to obtain numerical results exactly coinciding with the experimental results because of the extreme sensitivity of the system to the small parameter variations. However, the spectra presented in fig. 5 are in good qualitative agreement with the experimental spectra (figs. 2d–2f) corresponding to the "Chaos 2" regime.

In conclusion, we have verified experimentally the recently proposed method [9] for continuous control of chaos. The method does not require either a computer analysis of the system or computer control. A special analogue control circuit has been constructed. While in electronics the delayed feedback is usually associated with various instabilities, the considered control circuit is an example of how the delay can stabilize the system. The experimental realization of this method is very simple, particularly for high frequency systems, and we believe it can be applied to a variety of chaotic dynamical systems.

The authors acknowledge R.P. Huebener, A. Kittel, J. Parisi, J. Peinke, R. Richter and O.E. Rössler for stimulating discussion and N. Namajunas for the help in the experiment. This research was partially supported by the Alexander von Humboldt Foundation.

References

[1] E. Ott, C. Grebogi and J.A. Yorke, Phys. Rev. Lett. 64 (1990) 1196.
[2] U. Dressler and G. Nitsche, Phys. Rev. Lett. 68 (1992) 1.
[3] F.J. Romeiras, C. Grebogi, E. Ott and W.P. Dayawansa, Physica D 58 (1992) 165.
[4] D. Auerbach, C. Grebogi, E. Ott and J.A. Yorke, Phys. Rev. Lett. 69 (1992) 3479.
[5] W.L. Ditto, S.N. Rauseo and M.L. Spano, Phys. Rev. Lett. 65 (1990) 3211.
[6] A. Azevedo and S.M. Rezende, Phys. Rev. Lett. 66 (1991) 1342.
[7] E.R. Hunt, Phys. Rev. Lett. 67 (1991) 1953.
[8] R. Roy, T.M. Murphy Jr., T.D. Maier, Z. Gills and E.R. Hunt, Phys. Rev. Lett. 68 (1992) 1259.
[9] K. Pyragas, Phys. Lett. A 170 (1992) 421.
[10] J. Testa, Phys. Lett. A 111 (1985) 243.
[11] A. Namajūnas, J. Požela and A. Tamaševičius, Phys. Lett. A 131 (1988) 85.