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Predictable chaos in slightly perturbed unpredictable chaotic systems

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A method for stabilizing aperiodic orbits of a strange attractor is suggested. It enables the transformation of an unpredictable chaos into a predictable one by synchronizing the current behavior of a chaotic system with its past behavior. This is achieved by a *small* self-controlling feedback perturbation using the past output signal of the system, recorded previously in a memory. An experimental realization of the method is very simple. It does not require any computer analysis of the system behavior, and can be carried out by a purely analogous technique.

1. Introduction

It is well known that the prediction of the long-term behavior of chaotic systems is practically impossible, although these systems can be described by strongly determined dynamic models. Lorenz was the first to run into this problem when investigating the simple dynamic model consisting of three nonlinear ordinary differential equations [1]. The actual source of unpredictability is the property of a nonlinear system to separate initially close trajectories by an exponential law. Since, in practice, one can only fix the initial conditions of the system with finite accuracy, the errors increase exponentially fast. The characteristic time of reliable prediction is determined by the reciprocal of the maximal positive Lyapunov exponent of the system. Lorenz called this sensitive dependence on initial conditions the "butterfly effect", because the outcome of his equations, which describe in a crude sense the problem of weather forecasting, could be changed by a butterfly flapping its wings.

In spite of this fundamental difficulty, many investigations in the field of dynamic chaos are devoted to the development of forecasting methods [2–7]. These are based on building mathematical models directly from experimental data. The short-term prediction is then obtained as a solution of these models. It is common for all methods of forecasting to assume that the investigator (forecaster) is a passive subject, who cannot act on the system. The aim of this paper is to show that using only a *small* external perturbation of a special form, one can synchronize the current behavior of the system with its past behavior recorded previously in a memory. As a result, a reliable prediction becomes possible for any length of time.

The method suggested is based on stabilizing aperiodic orbits of the strange attractor. It represents a connection of two ideas, namely, the controlling chaos [8] suggested by Ott, Grebogi and Yorke (OGY), and the synchronization of chaos [9,10] suggested by Pecora and Carroll. OGY have suggested a method to stabilize the unstable *periodic* orbits of the strange attractor by using only a *small* feedback perturbation. The idea of Pecora and Carroll is based on synchronizing *aperiodic* orbits of two *strongly* coupled chaotic systems. Here we demonstrate the possibility of the stabilization of *aperiodic* orbits by a *small* feedback perturbation.

The subject of controlling chaotic systems has recently received a fair amount of attention of both

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theoretical [11–17] and experimental workers [18–22]. The standard methods of stabilizing periodic orbits are discrete in time since they deal with the Poincaré map of the system. The controlled perturbation is usually applied to the system one time per period. Recently we have proposed two methods of permanent control by a small self-controlling feedback [17]. They are noise resistant and can be realized in an experiment by an analogous technique. Here we extend the ideas of these methods for the case of stabilizing aperiodic orbits.

The paper is organized as follows. The method and its illustration for the Rössler [23], Lorenz [1], and Duffing [24] systems are presented in section 2. In section 3, the problem of stabilizing aperiodic orbits is reduced to the problem of synchronizing two identical chaotic systems. This permits the use of the conditional Lyapunov exponents, introduced by Pecora and Carroll as a criterion of stabilization. The influence of restricting the perturbation on the system transient dynamics is considered in section 4, and the conclusions are presented in section 5.

2. Method

Let us consider the chaotic system that can be simulated by a set of ordinary differential equations [17],

$$\dot{y}=P(y,x)+F(t), \quad \dot{x}=Q(y,x) . \tag{1}$$

We imagine that eqs. (1) are unknown, but some scalar variable $y(t)$ can be measured as a system output. The vector $x(t)$ describes the remaining variables of the system that are not available or are not of interest for observation. $F(t)$ is an external perturbation fed to the system input. Here we assume, for simplicity, that the input signal $F(t)$ disturbs only the first equation corresponding to the output variable. A more complicated multi-variable perturbation will be considered in section 3. The block diagram of the method is presented in fig. 1. The experiment is carried out in two stages. In the first, preparatory, stage an appropriate segment of the output signal $y_{ap}(t)$ of the unpertubed system has to be singled out and recorded in a memory. In the second stage, the system can be forced to repeat exactly

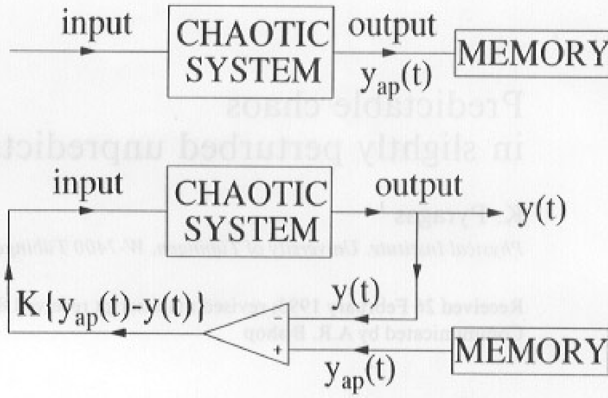

Fig. 1. Block diagram of the method.

the recorded signal by using a small feedback perturbation of the form

$$F(t)=K[y_{ap}(t)-y(t)] . \tag{2}$$

Here K is an experimentally adjustable weight of the perturbation. The perturbation has to be introduced into the system as a negative feedback ($K>0$). The important feature of this perturbation is that it vanishes when the output signal coincides with the signal recorded in a memory, $F(t)=0$ at $y(t)=y_{ap}(t)$. Therefore, it does not change the solution of the system corresponding to the segment of the aperiodic signal $y_{ap}(t)$. The perturbation performs the function of self-control, since it always tends to attract the current trajectory $y(t)$ of the system to the desired aperiodic orbit $y_{ap}(t)$. At a sufficiently large weight K, it can stabilize this trajectory. When the stabilization is achieved $y(t)\approx y_{ap}(t)$, and the perturbation becomes very small.

The results of such a stabilization for the Rössler, Lorenz, and Duffing systems are shown in fig. 2. After switching on the control, the perturbation is at first large, but then rapidly decreases to a very small value [#1]. After this transient process, the system begins to repeat exactly its previous behavior corre-

[#1] We have tried many different initial conditions for the trajectories $y(t)$ and $y_{ap}(t)$. The stabilization has been achieved for all systems considered independent of these conditions. Therefore, we ignore the possibility of other basins of attraction for now. Should this problem arise for some systems, we hope that it can be solved by restriction of the perturbation [17].

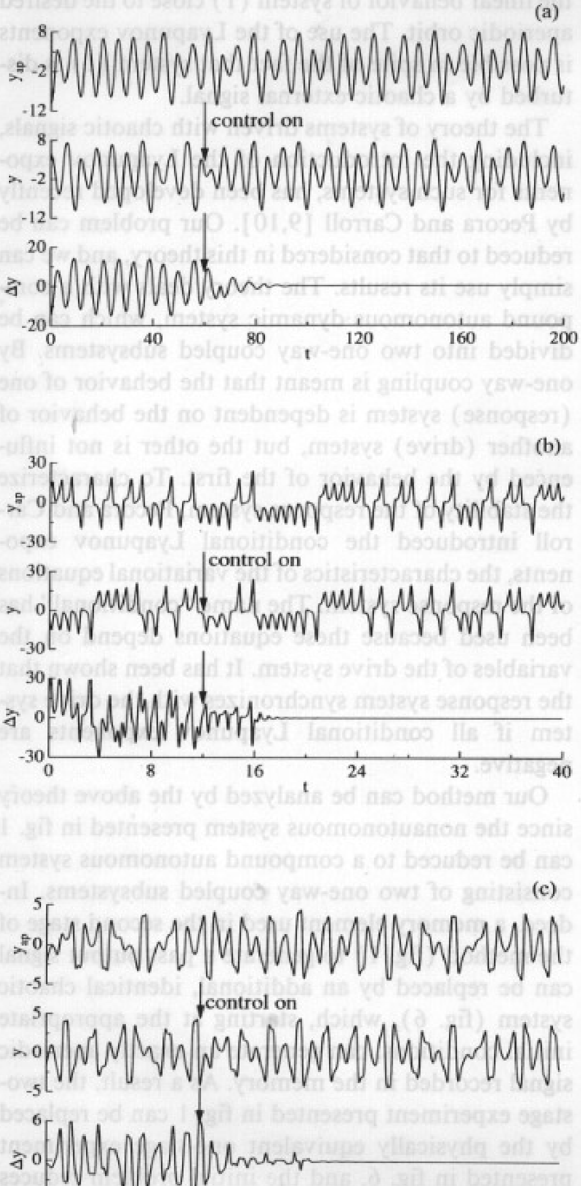

Fig. 2. Segments of "recorded" aperiodic output signals $y_{ap}(t)$ and the dynamics of the output signals $y(t)$ and the differences $\Delta y(t) = y_{ap} - y$, (a) for the Rössler system: $\dot{x} = -y - z$, $\dot{y} = x + 0.2y + K[y_{ap}(t) - y]$, $\dot{z} = 0.2 + z(x - 5.7)$, $K = 0.4$, (b) for the Lorenz system: $\dot{x} = 10(y - z)$, $\dot{y} = -xz + 28x - y + K[y_{ap}(t) - y]$, $\dot{z} = xy - \frac{8}{3}z$, $K = 4$, and (c) for the nonautonomous Duffing oscillator: $\dot{x} = y$, $\dot{y} = x - x^3 - dy + a\cos(\omega t) + K[y_{ap}(t) - y]$, $a = 2.5$, $\omega = 1$, $d = 0.02$, $K = 0.4$. The arrows show the moment of switching on the perturbation.

sponding to the recorded signal $y_{ap}(t)$. Therefore, the *small* feedback perturbation in the form of the difference between the output signal and the signal recorded in a memory forces the system to behave chaotically, however, makes it absolutely predictable. The resulting behavior depends, within certain limits, on our desire. The point is that any one of the different segments $y_{ap}(t)$ can be stabilized [#2], and the choice can be made to achieve the best system performance among those segments.

In a real experiment, the control will be negatively affected by at least two factors: fluctuation noise and gradual deviation of the system parameters from their initial values. These factors lead to the finite amplitude of the perturbation in a post-transient regime. Figure 3 illustrates the influence of both factors on the dispersion $\langle F^2(t)\rangle$ of the perturbation for the Duffing system. The amplitude of the perturbation decreases linearly with the decrease of the noise amplitude, as well as with the decrease of the parameter deviation. If both factors are small, the stabilization of the aperiodic orbit can be achieved with a very

[#2] The length of the segment has to be larger than the characteristic length of the transient process.

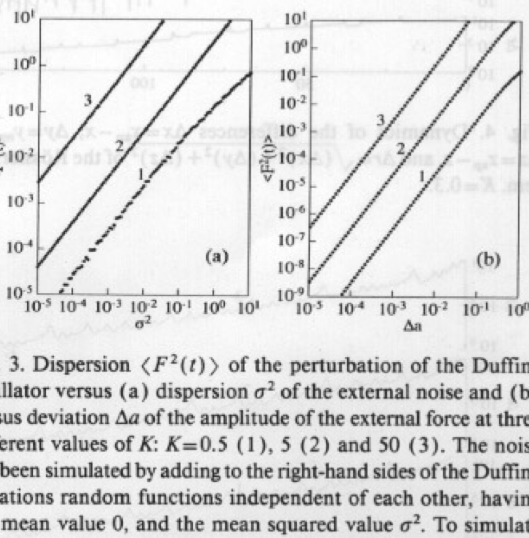

Fig. 3. Dispersion $\langle F^2(t)\rangle$ of the perturbation of the Duffing oscillator versus (a) dispersion σ^2 of the external noise and (b) versus deviation Δa of the amplitude of the external force at three different values of K: $K = 0.5$ (1), 5 (2) and 50 (3). The noise has been simulated by adding to the right-hand sides of the Duffing equations random functions independent of each other, having the mean value 0, and the mean squared value σ^2. To simulate the deviation of the amplitude a, we calculated at first an unperturbed aperiodic orbit $y_{ap}(t)$ at the fixed initial value $a = a_0 = 2.5$. Then the dynamics of the perturbed system has been calculated with the changed value $a = a_0 + \Delta a$, but with the old function $y_{ap}(t)$ corresponding to $a = a_0$.

small perturbation $F(t)$, and the experiment can be performed with a small external signal.

3. Linear analysis

In order to illustrate the law by which the perturbed system approaches the desired aperiodic orbit, fig. 4 shows the dynamics for the Rössler system in a half-logarithmic scale. As is evident from the figure, an asymptotic behavior follows an exponential law. The characteristic exponent depends neither on the initial conditions of the desired aperiodic orbit nor on the current initial conditions of the system corresponding to the moment of switching on the perturbation. This is illustrated in fig. 5 for the Lorenz system. The features above permit the introduction of the Lyapunov exponents as characteristics of

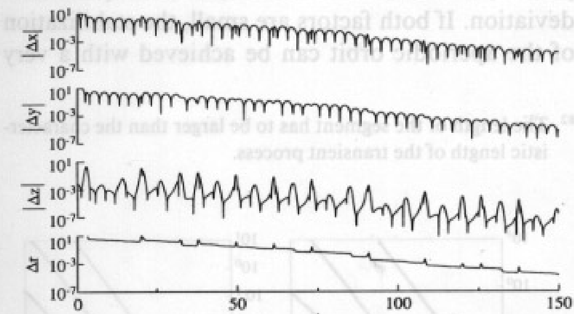

Fig. 4. Dynamics of the differences $\Delta x=x_{ap}-x$, $\Delta y=y_{ap}-y$, $\Delta z=z_{ap}-z$, and $\Delta r=\sqrt{(\Delta x)^2+(\Delta y)^2+(\Delta z)^2}$ of the Rössler system. $K=0.3$.

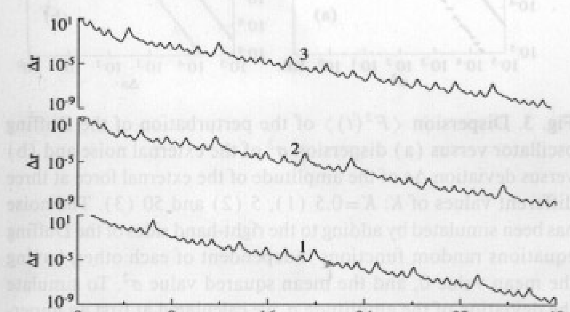

Fig. 5. Dynamics of the difference Δr of the Lorenz system at three different arbitrary chosen initial conditions for the signals $y_{ap}(t)$ and $y(t)$. $K=4$.

the linear behavior of system (1) close to the desired aperiodic orbit. The use of the Lyapunov exponents is possible in spite of the fact that system (1) is disturbed by a chaotic external signal.

The theory of systems driven with chaotic signals, including the introduction of the Lyapunov exponents for such systems, has been developed recently by Pecora and Carroll [9,10]. Our problem can be reduced to that considered in this theory, and we can simply use its results. The theory deals with a compound autonomous dynamic system, which can be divided into two one-way coupled subsystems. By one-way coupling is meant that the behavior of one (response) system is dependent on the behavior of another (drive) system, but the other is not influenced by the behavior of the first. To characterize the stability of the response system, Pecora and Carroll introduced the conditional Lyapunov exponents, the characteristics of the variational equations of the response system. The name "conditional" has been used because these equations depend on the variables of the drive system. It has been shown that the response system synchronizes with the drive system if all conditional Lyapunov exponents are negative.

Our method can be analyzed by the above theory since the nonautonomous system presented in fig. 1 can be reduced to a compound autonomous system consisting of two one-way coupled subsystems. Indeed, a memory element used in the second stage of the method (fig. 1) to generate a past output signal can be replaced by an additional, identical chaotic system (fig. 6), which, starting at the appropriate initial conditions, can generate an exactly aperiodic signal recorded in the memory. As a result, the two-stage experiment presented in fig. 1 can be replaced by the physically equivalent one-stage experiment presented in fig. 6, and the initial problem reduces to the problem of synchronizing two coupled, identical chaotic systems. Mathematically, this problem can be presented as follows [#3],

[#3] This presentation is also more convenient than (1), (2) for computer simulation. The difficulty with the application of higher-order Runge–Kutta methods to system (1), (2) is related to the fact that these methods require knowledge of the external signal $y_{ap}(t)$ values at the moments sited inside the integration intervals. This difficulty does not occur for system (3) since it is autonomous.

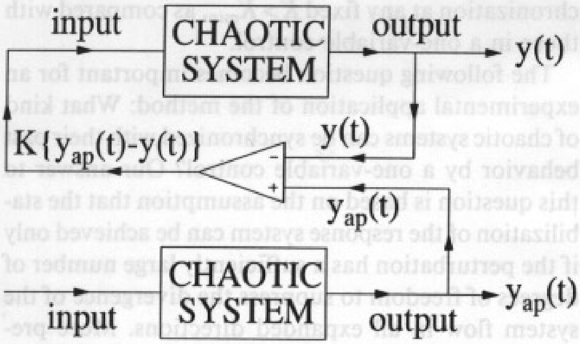

Fig. 6. Block diagram of a physically equivalent system to that presented in fig. 1. The memory element is replaced by an additional, identical chaotic system.

$$\text{drive:}\quad \dot y_{ap} = P(y_{ap}, x_{ap})\,,\qquad \dot x_{ap} = Q(y_{ap}, x_{ap})\,,$$

$$\text{response:}\quad \dot y = P(y, x) + K(y_{ap} - y)\,,\qquad \dot x = Q(y, x)\,. \tag{3}$$

The conditional Lyapunov exponents $\lambda(K)$ are defined by variational equations of the response system:

$$\delta\dot y = \delta y\frac{\partial}{\partial y}P(y_{ap}, x_{ap}) + \delta x\frac{\partial}{\partial x}P(y_{ap}, x_{ap}) - K\delta y\,,$$

$$\delta\dot x = \delta y\frac{\partial}{\partial y}Q(y_{ap}, x_{ap}) + \delta x\frac{\partial}{\partial x}Q(y_{ap}, x_{ap})\,. \tag{4}$$

Here $\delta y = y - y_{ap}$, $\delta x = x - x_{ap}$ define the deviations of the response system from the aperiodic orbit, determined by the drive system. Equations (4) differ from the variational equations defining the usual Lyapunov exponents of the unperturbed ($K=0$) system (1) by the term $-K\delta y$. At $K=0$, the conditional Lyapunov exponents coincide with the usual Lyapunov exponents of the unperturbed system. With the increase of K, this term results in a decrease of $\lambda(K)$ and the inversion of the sign of the initially positive Lyapunov exponents. Figure 7 shows the dependence of the maximal conditional Lyapunov exponents on K for the Rössler, Lorenz, and Duffing systems. The Lyapunov exponents are shown for all possible cases of a one-variable control and also for a multi-variable control. To explain these different cases, let us represent system (1) in a symmetrical form,

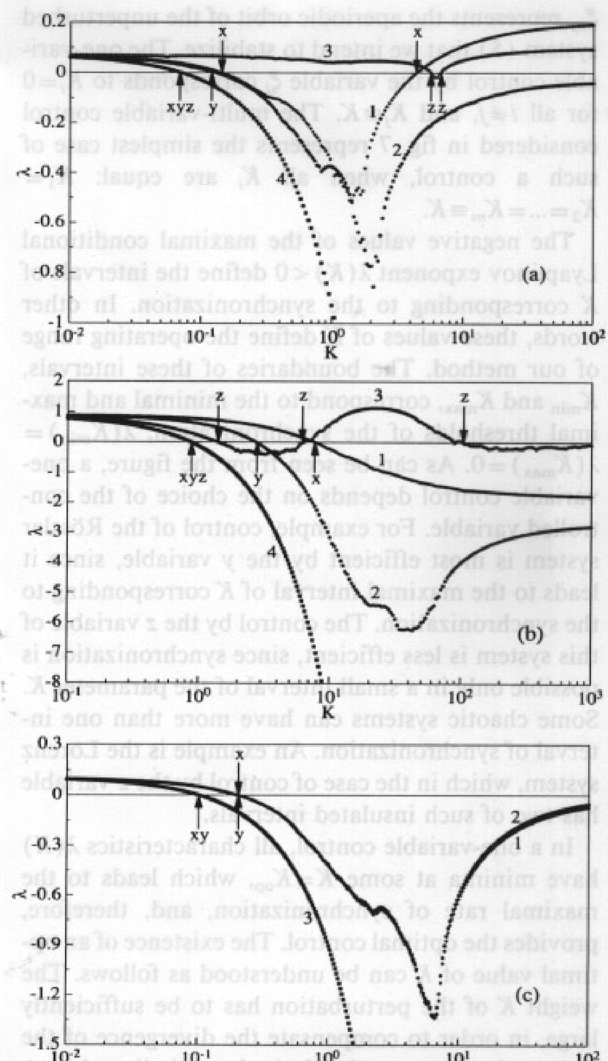

Fig. 7. Maximal conditional Lyapunov exponents λ versus weight K of the perturbation for the (a) Rössler (b) Lorenz, and (c) Duffing systems. Curves 1, 2, and 3 in diagrams (a) and (b) correspond to a one-variable control by the x, y, and z variable, respectively. Curve 4 corresponds to a multi-variable control: $K_x = K_y = K_z \equiv K$. In diagram (c) curves 1 and 2 correspond to a one-variable control by the x and y variables, respectively, and curve 3 corresponds to a multi-variable control: $K_x = K_y \equiv K$. The arrows, marked by corresponding controlled variables, show the thresholds of synchronization.

$$\dot\xi_i = \Phi_i(\xi) + K_i[(\xi_{ap})_i - \xi_i]\,,\qquad i = 1, 2, ..., m\,. \tag{5}$$

Here $\boldsymbol{\xi} \equiv \{\xi_1, \xi_2, ..., \xi_m\} \equiv \{y, x\}$ is the complete vector of the dynamic variables, $\boldsymbol{\Phi} = \{P, Q\}$, and the vector

ξ_{ap} represents the aperiodic orbit of the unperturbed system (5) that we intend to stabilize. The one-variable control by the variable ξ_j corresponds to $K_i=0$ for all $i\neq j$, and $K_j\equiv K$. The multi-variable control considered in fig. 7 represents the simplest case of such a control, when all K_i are equal: $K_1=K_2=...=K_m\equiv K$.

The negative values of the maximal conditional Lyapunov exponent $\lambda(K)<0$ define the intervals of K corresponding to the synchronization. In other words, these values of K define the operating range of our method. The boundaries of these intervals, K_{min} and K_{max}, correspond to the minimal and maximal thresholds of the synchronization, $\lambda(K_{min})=\lambda(K_{max})=0$. As can be seen from the figure, a one-variable control depends on the choice of the controlled variable. For example, control of the Rössler system is most efficient by the y variable, since it leads to the maximal interval of K corresponding to the synchronization. The control by the z variable of this system is less efficient, since synchronization is possible only in a small interval of the parameter K. Some chaotic systems can have more than one interval of synchronization. An example is the Lorenz system, which in the case of control by the z variable has two of such insulated intervals.

In a one-variable control, all characteristics $\lambda(K)$ have minima at some $K=K_{op}$, which leads to the maximal rate of synchronization, and, therefore, provides the optimal control. The existence of an optimal value of K can be understood as follows. The weight K of the perturbation has to be sufficiently large, in order to compensate the divergence of the trajectories close to the desired aperiodic orbit. A rather large K is not efficient since the perturbation disturbs only one equation of the system, corresponding to the output variable. For large K, the changes of this variable are very fast, and the remaining variables have no time to follow these changes. Therefore, one can conclude that the minimum in $\lambda(K)$ is related to the nonsymmetrical nature of a one-variable control. The calculation of $\lambda(K)$ in the case of multi-variable control supports this statement. This control leads to monotonically decreasing characteristics $\lambda(K)$ at any K, for all systems considered in this paper (fig. 7). Therefore, a multi-variable control is more efficient. It leads to a smaller threshold K_{min} and to a faster rate of synchronization at any fixed $K>K_{min}$, as compared with those in a one-variable control.

The following question becomes important for an experimental application of the method: What kind of chaotic systems can be synchronized with their past behavior by a one-variable control? Our answer to this question is based on the assumption that the stabilization of the response system can be achieved only if the perturbation has a sufficiently large number of degrees of freedom to suppress the divergence of the system flow in all expanded directions. More precisely, we assume that the minimal number of controlled variables has to be equal to the number of positive Lyapunov exponents of the system. All models considered up to now support this assumption. They all have only one positive Lyapunov exponent and they all can be synchronized by a one-variable control. To check this assumption for a more complicated system, we have considered the hyperchaos equations [25] with two positive Lyapunov exponents. The dependence of the conditional Lyapunov exponents on K for different types of control is shown in fig. 8. It is impossible to synchronize this system by a one-variable control: the maximal $\lambda(K)$ is positive at any K for all dynamic variables. However, it is possible to invert the sign of one out of two initially positive Lyapunov exponents. This is illustrated in the figure for the case of control by the y

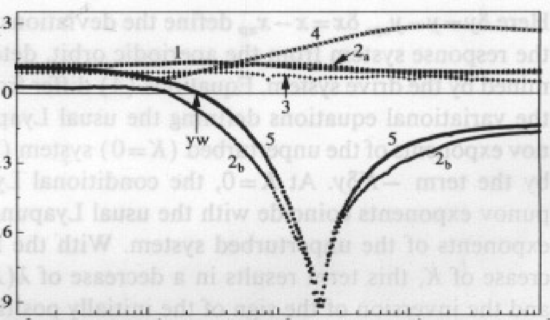

Fig. 8. Conditional Lyapunov exponents versus K for a hyperchaos system [25]: $\dot{x}=-y-z$, $\dot{y}=x+0.25y+w$, $\dot{z}=3+xz$, $\dot{w}=-0.5z+0.05w$. Curves 1, 2a, 3, and 4 correspond to the maximal Lyapunov exponents of a one-variable control by the x, y, z, and w variables, respectively. Curve 2b shows the second largest Lyapunov exponent in the case of control by the y variable. Curve 5 corresponds to a two-variable control by the y and w variables simultaneously: $K_y=K_w\equiv K$.

variable. Although synchronization is impossible here, the flow of the dynamic system close to the desired aperiodic orbit diverges now only in one unstable direction. One can say that close to this orbit the control turns hyperchaos into chaos. Applying the perturbation to two equations of the system, one can invert the sign of both positive Lyapunov exponents. This is illustrated in the figure for the case of a two-variable control by the y and w variables. Therefore, this model also supports the above assumption.

4. Restriction of the perturbation

Let us discuss now the transient process. The initial amplitude of the perturbation depends on the distance between the states of the response and the drive systems at the moment of switching on the perturbation. In the typical case, this distance is not short, and the perturbation has a large initial amplitude. Large initial values of the perturbation can be undesired or inaccessible for some experimental situations. Here, as well as in our previous paper [17], we consider the restricted perturbation of the form

$$\begin{aligned} F(t) &= -F_0\,, & KD(t) &\leqslant -F_0\,, \\ &= KD(t)\,, & -F_0 &< KD(t) < F_0\,, \\ &= F_0\,, & KD(t) &\geqslant F_0\,. \end{aligned} \tag{6}$$

Here $F_0>0$ is the saturating value of the perturbation, and $D(t)=y_{\text{ap}}(t)-y(t)$. Saturation can be achieved by introducing some nonlinear element into the feedback circuit. In proximity to the recorded signal, $y(t)\approx y_{\text{ap}}(t)$, both perturbations (2) and (6) are working identically, but they are leading to different transients. Figure 9 illustrates the influence of the restriction on the system dynamics. Here the perturbation is always small including the transient process, however, the duration of this process, on average, is now much longer. The control is not sufficiently efficient until the state of the response system does not come close to the state of the drive system.

For small F_0, the average time of the transient $\langle \tau_0 \rangle$ can be estimated as follows. The probability of the

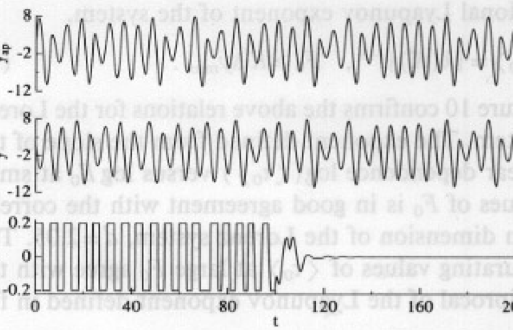

Fig. 9. The same as in fig. 2a, but for the case of a restricted perturbation. $F_0=0.2$. The perturbation is switched on at $t=0$.

repetition of the state of the dynamic system with some accuracy ϵ is proportional to the correlation integral $C_m(\epsilon)$ that scales as $C_m(\epsilon)\propto\epsilon^d$ [26]. Here d is the correlation dimension of the strange attractor. The efficient control leading to the synchronization is possible only if the difference ϵ of the states is of the order of the amplitude of the perturbation, $\epsilon\propto F_0$. Therefore, the average time $\langle\tau_0\rangle$ of the transient increases with the decrease of F_0 by a power law,

$$\langle\tau_0\rangle \propto C_m^{-1}(\epsilon)\propto F_0^{-d}\,, \quad F_0 \ll K\Delta y_{\max}\,. \tag{7}$$

Here $\Delta y_{\max}$ is the size of the strange attractor in the y direction.

For large $F_0>K\Delta y_{\max}$, the perturbation does not achieve the saturating value F_0, and the system behaves in the same manner as if without any restriction. The average length of the transient $\langle\tau_0\rangle$ now is proportional to the reciprocal of the maximal con-

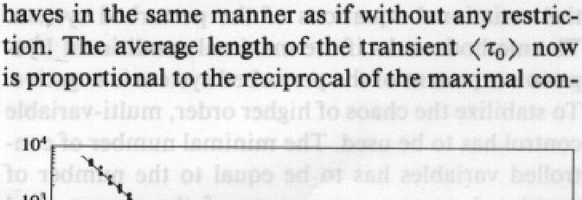

Fig. 10. The average time $\langle\tau_0\rangle$, needed to achieve synchronization of the Lorenz system, versus maximally allowed amplitude of the perturbation F_0 for three different values of K: $K=4$ (1), 10 (2) and 40 (3).

ditional Lyapunov exponent of the system,

$$\langle \tau_0 \rangle = |\lambda(K)|^{-1}, \quad F_0 \geqslant K \Delta y_{\max} . \qquad (8)$$

Figure 10 confirms the above relations for the Lorenz system. The exponent defined from the slope of the linear dependence $\log(\langle \tau_0 \rangle)$ versus $\log F_0$ at small values of F_0 is in good agreement with the correlation dimension of the Lorenz system, $d=2.05$. The saturating values of $\langle \tau_0 \rangle$ at large F_0 agree with the reciprocal of the Lyapunov exponent defined in fig. 7b.

5. Conclusions

We have shown that the current behavior of a chaotic system can be synchronized with its past behavior, recorded previously in a memory. This is achieved by a small self-controlling feedback perturbation in the form of the difference between the current and past output signals. As a result, the system behavior becomes absolutely predictable. This behavior can be changed, within certain limits, according to our desire by choosing different intervals of the past output signal. An experimental application of the method does not require any computer analysis of the system behavior. It can be easily carried out by a purely analogous technique. The operating range of the method can be determined from the variational equations of the perturbed system. The method works if the maximal conditional Lyapunov exponent of the perturbed system is negative. To stabilize the chaos of higher order, multi-variable control has to be used. The minimal number of controlled variables has to be equal to the number of positive Lyapunov exponents of the unperturbed system.

The large initial values of the perturbation corresponding to the transient dynamics can be avoided by some restriction of the perturbation. However, the transient process becomes longer in this case. The average length of the transient increases with the decrease of the maximally allowed amplitude of the perturbation by a power law with the characteristic exponent being equal to the dimension of the strange attractor.

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